Conference on Mathematics of Wave Phenomena

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Book of Abstracts

Conference on Mathematics of Wave Phenomena

held at the Department of Mathematics, Karlsruhe Institute of Technology, Germany, July 23–27, 2018

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wave phenomena
analysis and numerics

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Preface

The mathematical modeling, simulation and analysis of wave phenomena entail a plethora of fascinating and challenging problems both in analysis and numerical mathematics. During the past decades, these challenges have inspired a number of important approaches, developments, and results about wave-type equations. This conference brings together international experts with different background to stimulate the transfer of ideas, results, and techniques within this exciting area.

The scientific programme of the conference consists of eleven plenary lectures, contributions in fourteen minisymposia as well close to fifty contributed talks. This book contains summaries of all these contributions. We start with the plenary lectures, followed by minisymposia and finally other contributed talks. These presentations span the whole width of the field of analytical or numerical studies of wave phenomena. Subjects include non-linear wave equations, waves in waveguides, water waves, waves in non-linear media, direct and inverse scattering problems for acoustic, electro-magnetic or elastic waves, finite and boundary element methods, and both time-harmonic and transient wave propagation problems.

The conference is organized at Karlsruhe Institute of Technology by members of the Collaborative Research Centre 1173 (CRC 1173) on Wave Phenomena. We gratefully acknowledge support from the Deutsche Forschungsgemeinschaft (DFG) through CRC 1173.

Karlsruhe, June 2018

Tilo Arens
Willy Dörfler
Tanja Hagedorn
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Plenary Lectures

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Plenary Talk **Monday, 09:15-10:15, Tulla Lecture Theatre (11.40)**

Two dimensional water waves and related models

Daniel Tataru[1,*]

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The aim of this talk will be to provide an overview of recent work concerned with the long time dynamics for several two dimensional water wave type evolutions. In addition, the relation with several one dimensional integrable model problems will also be discussed. This is all joint work with Mihaela Ifrim, and in part with several other collaborators.

Plenary Talk **Monday, 10:15-11:15, Tulla Lecture Theatre (11.40)**

Linearized wave turbulence for three-wave systems

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The goal of this talk is to present some recent results contributing to a mathematical analysis of wave turbulence, *i.e.* the statistical analysis of solutions to weakly nonlinear wave equations submitted to external forcing and dissipation.

We shall consider stochastic and deterministic three-wave semi-linear systems of KP type (Kadomtsev-Petviashvili), set on large periodic domains. We assume that the nonlinearity is small and acting only on a bounded set of frequencies, and that the noise is small or void and acting only in the angles of the Fourier modes (random phase forcing). We consider random initial data and show that these systems possess natural invariant distributions corresponding to some Rayleigh-Jeans stationary solutions of the wave kinetic equation appearing in wave turbulence theory.

We then consider random initial modes drawn with probability laws that are perturbations of theses invariant distributions. In the stochastic case, we prove that in the asymptotic limit (small nonlinearity, continuous set of frequency and small noise), the renormalized fluctuations of the amplitudes of the Fourier modes converge in a weak sense towards the solution of the *linearized wave kinetic equation* around these Rayleigh-Jeans spectra. Moreover, we show that in absence of noise, the deterministic equation with the same random initial satisfies a generic Birkhoff reduction in a probabilistic sense, without kinetic description at least in some regime of parameters.

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Plenary Talk **Tuesday, 09:00-10:00, Tulla Lecture Theatre (11.40)**

Energy concentration and singularity formation in nonlinear wave dynamics

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Energy concentration during the propagation of nonlinear waves is expected in a wide range of physical systems, from astrophysics to fluid mechanics, but is still very poorly understood both at the formal and rigorous level. The question of the existence of incompressible fluid singularities is one the challenging Clay problems. I will review on this talk some recent results concerning the possibility of energy concentration and singularity formation for canonical wave models like the nonlinear Schrodinger equation. I will in particular explain how solitons play a distinguished role in the the singularity formation mechanisms, both in sub critical and energy super critical regimes.

Plenary Talk **Tuesday, 10:00-11:00, Tulla Lecture Theatre (11.40)**

Trefftz Approximations for Complex Structures

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Trefftz approximations, by definition, involve functions that satisfy (locally) the underlying differential equation of a given problem, along with the relevant interface boundary conditions. Simple examples include harmonic polynomials for the Laplace equation; plane waves, cylindrical or spherical harmonics for wave problems, and so on. At the same time, more complex cases are of great theoretical and practical interest: waves around particles or other inhomogeneous objects, Bloch modes in periodic media, and much more.

The talk has three main parts. The first two focus on two particular applications of Trefftz functions in electromagnetic analysis: (i) non-asymptotic / nonlocal homogenization of periodic structures [1,2], and (ii) high-order Trefftz-based difference schemes for electromagnetic scattering [3].

The third part addresses a key question: what explains the "unreasonable effectiveness" of Trefftz approximations (to paraphrase Eugene Wigner)? In the mathematical literature, this question has been studied primarily for homogeneous media [4–9] (e.g. plane wave or cylindrical wave expansions) but needs to be posed much more broadly [10]. There is, in particular, an interesting connection with the theory of random matrices [10, 11].

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Plenary Talk Wednesday, 09:00-10:00, Tulla Lecture Theatre (11.40)

A new complex frequency spectrum for the analysis of tranmission properties in perturbed waveguides

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[2] *INRIA, Ecole Polytechnique, Palaiseau, France*
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We consider here acoustic waveguides with a bounded cross-section, where only a finite number of modes can propagate at a given frequency, other modes being evanescent. If an infinite waveguide is locally perturbed, a superposition of incident propagating modes will generally produce a superposition of reflected propagating modes. As a consequence, only a part of the total incident energy will be transmitted from the inlet to the outlet. But at some exceptional frequencies and for particular incident waves, it may occur that 100% of the energy is transmitted, the only effect in reflection being a superposition of evanescent modes in the vicinity of the perturbation. An important question for the applications is to identify these reflection-less frequencies.

A first expensive way to do that is to compute the scattering matrix for all frequencies in the range of interest. For numerical purpose, the selection of the outgoing scattered wave can be achieved by using Perfectly Matched Layers (PML) on each side of the obstacle (see [1]), which amounts to introducing a complex dilation of the scattered field in the axial coordinate, with some complex coefficient α. It is well-known that this problem is ill-posed for a discrete family of eigenfrequencies: the ones on the real axis correspond to the trapped modes and the others to the complex resonances [2].

We have proposed in [3] an alternative approach for which reflection-less frequencies appear directly as eigenvalues of a new problem. This problem is very similar to the previous one, except a slight modification in the PML. Precisely, we use two conjugated dilation parameters, α in the outlet and $\bar{\alpha}$ in the inlet, in order to select outgoing waves in the outlet and ingoing waves in the inlet. The important idea behind that is the following: if the total field is ingoing in the inlet, then it means precisely that there are no reflection, as wanted. In fact, we show that the real eigenvalues that are obtained correspond either to trapped modes (mentionned above) or to reflection-less modes. In addition to this real spectrum, we find intrinsic complex frequencies, which also contain information about the quality of the transmission through the waveguide.

Mathematically, the non-selfadjoint eigenvalue problem with conjugated PMLs has strange properties: the essential spectrum divides the complex plane in a countable family of connected components. As a consequence, the discreteness of the point spectrum is not stable by compact perturbations and pathological examples can be exhibited.

Finally, let us mention that the approach also applies to the junction of several semi-infinite waveguides, and to other types of waves (electromagnetic waves, elastodynamic waves or water waves). Several illustrations will be presented.

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Plenary Talk **Wednesday, 10:00-11:00, Tulla Lecture Theatre (11.40)**

Self-similar blowup in supercritical wave equations

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Many supercritical wave equations exhibit finite-time blowup via self-similar solutions. The relevance for the generic Cauchy problem of such explicit examples of singular solutions depends on their stability. In the last ten years the rigorous understanding of stability properties of self-similar blowup improved considerably. I will review some recent resuls in this direction.

Plenary Talk **Thursday, 09:00-10:00, Tulla Lecture Theatre (11.40)**

Nonlinear processing of active array data in inverse scattering via reduced order models

Liliana Borcea[1,*], **Vladimir Druskin**[2], **Alexander Mamonov**[3], **Mikhail Zaslavsky**[4]
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[2]*Schlumberger Doll-Research Center, Cambridge, Massachussets*
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We discuss an inverse problem for the wave equation, where an array of sensors probes an unknown, heterogeneous medium with pulses and measures the scattered waves. The goal in inversion is to determine from these measurements scattering structures in the medium, modeled mathematically by a reflectivity function. Most imaging methods assume a linear mapping between the unknown reflectivity and the array data. The linearization, known as the Born (single scattering) approximation is not accurate in strongly scattering media, so the reconstruction of the reflectivity may be poor. We show that it is possible to remove the multiple scattering (nonlinear) effects from the data using a reduced order model (ROM). The ROM is defined by an orthogonal projection of the wave propagator operator on the subspace spanned by the time snapshots of the solution of the wave equation. The snapshots are known only at the sensor locations, which is enough information to construct the ROM. The main result discussed in the talk is a novel, linear-algebraic algorithm that uses the ROM to map the data to its Born approximation. I will discuss how it applies to imaging with sound, electromagnetic and elastic waves.

References

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Plenary Talk **Thursday, 10:00-11:00, Tulla Lecture Theatre (11.40)**

Effective description of waves in discrete and heterogeneous media

Ben Schweizer[1,*]
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Homogenization theory predicts that waves in periodic heterogeneous media can be described, in the limit of small periodicity, by an effective wave equation. This is true as long as finite observation times are considered, but it is no longer true for large time intervals, which are relevant in many applications. On large time intervals, one observes dispersion, which means that waves of different wave-length travel with different speed. In particular, a wave pulse will, in general, change its form in the course of time. A linear wave equation with constant coefficients does not show dispersion and cannot explain the observed effect. We must instead find a dispersive model. We showed that, effectively, a linear wave equation with periodic coefficients and with a small periodicity can be replaced, in a new homogenization limit, by a linear wave equation of fourth order with constant coefficients. The predictions of this weakly dispersive model agrees perfectly with numerical results. We furthermore investigate the wave equation in a discrete spring-mass model. The discrete character of the model introduces small-scale oscillations, which result again in a dispersive long time behavior. We derive the dispersive effective wave equations also for the discrete model. Moreover, for ring-like solution fronts that occur for localized initial data after long time, we derive the equations that dictate the evolution of the front: Our derivation provides a linearized KdV equation and an explicit representation of the corresponding initial data in Fourier space. We present work that was obtained in collaborations with A. Lamacz, T. Dohnal, and F. Theil.

Plenary Talk **Friday, 09:00-10:00, Tulla Lecture Theatre (11.40)**

Nonlinear stability of sources

Margaret Beck[1], **Toan Nguyen**[2], **Björn Sandstede**[3,*], **Kevin Zumbrun**[4]
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[2]*Department of Mathematics, Pennsylvania State University, State College, USA*
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Defects are interfaces that mediate between two wave trains with possibly different wave numbers. Of particular interest in applications are sources for which the group velocities of the wave trains to either side of the defect point away from the interface. While sources are ubiquitous in experiments and can be found easily in numerical simulations of appropriate models, their stability analysis still presents many challenges. One difficulty is that sources are not travelling waves but are time-periodic in an appropriate moving coordinate frame. A second difficulty is that perturbations are transported towards infinity, which makes it difficult to apply various commonly used techniques. In this talk, I will discuss nonlinear stability results for sources in general reaction-diffusion system and outline a proof that utilizes pointwise estimates.

Plenary Talk **Friday, 10:00-11:00, Tulla Lecture Theatre (11.40)**

Optimal design of thin film solar cells

Peter Monk[1,*]
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The optimal design of thin film solar cells requires modeling of both 1) the absorption of solar light and 2) the flow of currents throughout the device. The goal is to maximize the efficiency of the device by ensuring the optical and electrical behaviors complement one another. Because of the presence of local maxima in the figure efficiency (our figure–of–merit), we choose to search for an optimal design using a population based stochastic search technique called the Differential Evolution Algorithm. This algorithm requires a large number of evaluations of the figure–of–merit functional to achieve a reliable estimate of the optimal state, but no derivative information is needed. In addition, the figure–of–merit may need to be evaluated for widely different device configurations. These requirements have influenced our modeling choices.

For the optical modeling, we need to solve Maxwell's equations across a wide range of wavelengths throughout the visible solar spectrum in order to predict the generation rate of free electrons. We use a Fourier based technique with a special solver, called the Rigorous Coupled Wave Analysis (RCWA) method. This method is fast and does not require remeshing between solar cells with different geometrical structures. The output from RCWA is used as a source term in the electrical model, driving the flow of electrons and holes in semiconductor elements of the device. This electronic behaviour is modeled using the classical drift-diffusion equations, which are approximated using the Hybridizable Discontinuous Galerkin (HDG) method. This formulation naturally allows upwinding and the handling of heterojunctions. I shall describe each component of the model, giving numerical results, and show how we can achieve a successful optimal design process.

Plenary Talk **Friday, 11:30-12:30, Tulla Lecture Theatre (11.40)**

Fast solvers for frequency domain wave propagation

Björn Engquist[1,*]
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Direct numerical approximation of high frequency wave propagation typically requires a very large number of unknowns and is computationally very costly. We will discuss two aspects of this type of problem formulated in frequency domain. One is the development and analysis of fast numerical algorithms of optimal computational complexity for boundary integral formulations and variable coefficient differential equations. In the variable coefficient case the challenge is preconditioning. The algorithms are based on separable approximation lemmas. The other aspect is analysis revealing when algorithms of this type of operator compression are possible and when they are not. The analysis is based on stationary phase and lower bounds for low rank approximations.

Minisymposia

List of Minisymposia

Minisymposium 1 **Tuesday, 11:30-12:00, Seminar room 2.067 (20.30)**

Branching processes representation of solutions to non-linear Dirac equations

Tomasz Zastawniak[1,*]

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For the solutions of the non-linear Dirac equation in two space-time dimensions, we establish Feynman-Kac type representations in terms of branching Poisson processes. These resemble McKean's (1975) branching diffusion representation for the solution to the non-linear reaction-diffusion equation of Kolmogorov-Petrovskii-Piskunov (the KPP equation). The representations in terms of branching Poisson processes are established for a wide range of polynomial non-linearities in the Dirac equation, including a number of cases studied in physics such as the Thirring model, the Gross-Neveu model, as well as the non-linear Dirac equations in the context of Feshbach resonance for Bose-Einstein condensates, in periodic dielectric materials under Bragg resonance, or in modelling wave resonances in photonic crystals. This is joint work with Zdzisław Brzeźniak, University of York.

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Minisymposium 1 **Tuesday, 12:00-12:30, Seminar room 2.067 (20.30)**

On the stochastic Gross-Pitaevskii equation

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In Physics, the stochastic Gross-Pitaevskii equation is used as a model to describe Bose-Einstein condensation at positive temperature. The equation is in fact a complex Ginzburg-Landau equation with a trapping potential and an additive space-time white noise. I am going to talk about two important questions and corresponding our results for this system: the global existence of solutions in the support of the Gibbs measure, and the convergence of those solutions to the equilibrium for large time.

Minisymposium 1 **Tuesday, 12:30-13:00, Seminar room 2.067 (20.30)**

On existence, uniqueness, regularity and invariant measures for stochastic wave equations

Martin Ondreját[1,*]

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We will survey some recent results on existence, uniqueness, regularity and invariant measures for non-linear stochastic wave equations with values in flat spaces and in Riemannian manifolds. The talk is partly based on several joint works with L. Baňas, Z. Brzeźniak, E. Motyl, M. Neklyudov, J. Seidler and A. Prohl.

Minisymposium 1 **Tuesday, 15:00-15:30, Seminar room 2.067 (20.30)**

Singular Stochastic PDEs for the Anderson Hamiltonian

Baris Evren Ugurcan[1,*]

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This talk is about well-posedness of singular dispersive stochastic PDEs, treated in the setting of paracontrolled distributions. I will report results on the cases of nonlinear Schrödinger and wave equations, with multiplicative noise, in 2d and 3d with periodic domain; and talk about existence, uniqueness and convergence of the regularized solutions with certain nonlinearities. I will also present some recent work regarding the full space case. Based on joint works with Gubinelli and Zachhuber.

Minisymposium 1 **Tuesday, 15:30-16:00, Seminar room 2.067 (20.30)**

Stochastic Strichartz estimates and the NLS with multiplicative noise

Fabian Hornung[1,*]
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In this talk, we consider the nonlinear Schrödinger equation

$$\begin{cases} \mathrm{d}u(t) = \left(\mathrm{i}\Delta u(t) - \mathrm{i}\lambda |u(t)|^{\alpha-1} u(t)\right) \mathrm{d}t - \displaystyle\sum_{m=1}^{\infty} e_m |u(t)|^{\gamma-1} u(t) \circ \mathrm{d}\beta_m(t), \\ u(0) = u_0, \end{cases}$$

with a multiplicative Stratonovich noise. We present global existence and uniqueness results for initial values $u_0 \in L^2(\mathbb{R}^d)$ and the full range of L^2-subcritical nonlinearities under weak assumptions on the regularity of the noise. The proof is based on deterministic and stochastic Strichartz estimates and the mass conservation of the stochastic NLS. Moreover, we sketch a similar reasoning for initial values in $H^1(\mathbb{R}^d)$.

Minisymposium 1 **Tuesday, 16:30-17:00, Seminar room 2.067 (20.30)**

Ergodicity of the Gibbs measure for the one dimensional stochastic cubic wave equation with damping

Leonardo Tolomeo[1,*]
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In this talk, we consider the Cauchy problem for the defocusing cubic stochastic damped wave equation (SNLB) with additive space-time white noise forcing, posed on the one dimensional torus. We study the long time behaviour of solutions this equation. Using techniques deriving from the study of dispersive equations with random initial data, we show existence of the flow and invariance of a suitable Gibbs-type measure. Thereafter, using the splitting of the flow into a linear part and a smoother remainder, we show ergodicity of this measure and convergence to equilibrium starting from any initial data.

Minisymposium 1 **Tuesday, 17:00-17:30, Seminar room 2.067 (20.30)**

Random-field Solutions of Weakly Hyperbolic Stochastic PDEs with Polynomially Bounded Coefficients

Alessia Ascanelli[1,*], **Sandro Coriasco**[2], **André Süß**

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We study random-field solutions of a class of linear stochastic partial differential equations, involving operators with polynomially bounded coefficients, under suitable hyperbolicity hypotheses. We provide conditions on the initial data and on the stochastic term, namely, on the associated spectral measure, so that such kind of solutions exist and are unique in suitably chosen functional classes. We also give a regularity result for the expected value of the solution.

References

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[2] A. Ascanelli, A. Süß, Random-field solutions to linear hyperbolic stochastic partial differential equations with variable coefficients, *Stochastic Processes and their Applications* (2017), DOI 10.1016/j.spa.2017.09.019.

Minisymposium 1 **Tuesday, 17:30-18:00, Seminar room 2.067 (20.30)**

Mild Solutions of Weakly Hyperbolic Semilinear SPDEs with Polynomially Bounded Coefficients

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We study function-valued solutions of a class of semilinear stochastic equations, associated with (polynomially bounded, variable coefficients) linear operators, satisfying suitable hyperbolicity hypotheses. We provide conditions on the initial data and on the stochastic term, namely, on the associated spectral measure, such that function-valued solutions uniquely exist, and a regularity result is obtained. A comparison with random-field solutions, in the case where the nonlinearity is trivial, is also performed.

References

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Minisymposium 2 Wednesday, 11:30-12:00, Seminar room 0.014 (20.30)

Linear Sampling Method applied to Non Destructive Testing of an elastic waveguide: experimental validation

Laurent Bourgeois[1,*], **Arnaud Recoquillay**[2], **Vahan Baronian**[2], **Bastien Chapuis**[2]
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In this lecture, we focus on an application of the Linear Sampling Method to ultrasonic inspection of an elastic waveguide in the presence of real data. In particular, those real data are surface data in the time domain, and are given on only one side of the defect we wish to detect (backscattering). The strategy we adopt consists in transforming the surface data in the time domain into modal data in the frequency domain. This allows us to discretize and regularize the ill-posed problem in a very physical way based on the propagating modes, which are the only guided modes that propagate at long distance. In addition, such method allows us to optimize the number and location of sources and receivers. It should be noted that our modal approach, in the context of elasticity, requires the introduction of special vector fields that mix some components of the displacement vector and some components of the stress tensor. Some experimental results will show that the Linear Sampling Method can be adapted to the context of Non Destructive Testing in real life. This lecture is based on [1].

References

[1] V. Baronian, L. Bourgeois, B. Chapuis and A. Recoquillay, to appear in Inverse Problems, *Linear Sampling Method applied to Non Destructive Testing of an elastic waveguide: theory, numerics and experiments*

Minisymposium 2 Wednesday, 12:00-12:30, Seminar room 0.014 (20.30)

Non reflection and perfect reflection via Fano resonance in waveguides

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We investigate a time-harmonic wave problem in a waveguide. By means of asymptotic analysis techniques, we justify the so-called Fano resonance phenomenon. More precisely, we show that the scattering matrix considered as a function of a geometrical parameter ε and of the frequency λ is in general not continuous at a point $(\varepsilon, \lambda) = (0, \lambda^0)$ where trapped modes exist. In particular, we prove that for a given $\varepsilon \neq 0$ small, the scattering matrix exhibits a rapid change for frequencies varying in a neighbourhood of λ^0. We use this property to construct examples of waveguides such that the energy of an incident wave propagating through the structure is perfectly transmitted (non reflection) or perfectly reflected in monomode regime. We provide numerical results to illustrate our theorems.

References

[1] L. Chesnel, S.A. Nazarov, Non reflection and perfect reflection via Fano resonance in waveguides, preprint arXiv:1801.08889, 2018.

Minisymposium 2 **Wednesday, 12:30-13:00, Seminar room 0.014 (20.30)**

Qualitative methods in terminating waveguide imaging

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We consider the inverse problem to reconstruct unknown perturbations (such as deformations of the wall or obstacles inside) in an acoustic terminating waveguide. Using the scattered field measured at the the opening of the terminating waveguide, we apply the qualitative methods (linear sampling method and factorization method) to reconstruct the perturbations. We further demonstrate that how the factorization method may lead to migration type method by the example of imaging an obstacle. We illustrate the feasibility of those qualitative methods by numerical examples. This work is in collaboration with Liliana Borcea (University of Michigan) and Fioralba Cakoni (Rutgers University).

Minisymposium 2 **Wednesday, 16:30-17:00, Seminar room 0.014 (20.30)**

Asymptotic expansions for transmission eigenvalues for media with small inhomogeneities

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We consider the transmission eigenvalue problem for an inhomogeneous media of compact support perturbed by small penetrable homogeneous inclusions. Assuming that the inhomogeneous background media is known and smooth, we investigate how these small volume inclusions affect the transmission eigenvalues. Our perturbation analysis makes use of the formulation of the transmission eigenvalue problem introduced by Kirsch, which requires that the contrast of the inhomogeneity is of one-sign only near the boundary. Thus, our approach can handle small perturbations with positive, negative or zero (voids) contrasts. In addition to proving the convergence rate for the eigenvalues corresponding to the perturbed media as inclusions' volume goes to zero, we also provide the explicit first correction term in the asymptotic expansion for simple eigenvalues. The correction term involves computable in- formation about the known inhomogeneity as well as the location, size and refractive index of small perturbations. Our asymptotic formula has the potential to be used to recover information about small inclusions from knowledge of the real transmission eigenvalues, which can be determined from scattering data.

Based on joint work with Fioralba Cakoni and Scott Rome.

Minisymposium 2	Wednesday, 17:00-17:30, Seminar room 0.014 (20.30)

Monotonicity in inverse medium scattering on unbounded domains

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We consider a time-harmonic inverse scattering problem for the Helmholtz equation with compactly supported penetrable and possibly inhomogeneous scattering objects in an unbounded homogeneous background medium, and we discuss a monotonicity relation for the far field operator that maps superpositions of incident plane waves to the far field patterns of the corresponding scattered waves. We utilize this monotonicity relation to establish novel characterizations of the support of the scattering objects in terms of the far field operator. These are related to and extend corresponding results known from factorization and linear sampling methods to determine the support of unknown scattering objects from far field observations of scattered fields. An attraction of the new characterizations is that they only require the refractive index of the scattering objects to be above or below the refractive index of the background medium locally and near the boundary of the scatterers. We present numerical examples to illustrate our theoretical findings.

Minisymposium 2	Wednesday, 17:30-18:00, Seminar room 0.014 (20.30)

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Analysis of sampling methods for locally perturbed periodic media using a single Floquet Bloch mode

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This work is concerned with the analysis and extension of the differential sampling method introduced in [1] to recover the support of a local perturbation in a periodic layer from measurements of scattered waves at a fixed frequency. The analysis is based on studying sampling methods using a single Floquet-Bloch mode. It is shown in [1] that the method is capable of directly reconstructing the perturbation (without knowing the period background physical properties) under restrictive assumption that the perturbation does not intersect with the periodic background. We extend these results by analyzing the general case where the perturbation is included or has a non-empty intersection with the periodic background. The main ingredient is the study of a new type of interior transmission problems that couple usual interior transmission problems for one Floquet-Bloch mode with scattering problems for the other modes. We shall discuss and analyze this problem as well as the sampling methods for differential imaging and present some related numerical results.

References

[1] Houssem Haddar and Thi-Phong Nguyen, Sampling methods for reconstructing the geometry of a local perturbation in unknown periodic layers, *Computers and Mathematics with Applications* **74** (2017), pp. 2831–2855.

Minisymposium 2 **Thursday, 11:30-12:00, Seminar room -1.025 (20.30)**

Imaging through random media by speckle intensity correlations

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When waves propagate through random media the energy is transferred to the incoherent wave part by scattering. The wave intensity then forms a random speckle pattern seemingly without much useful information. However, a number of recent physical experiments show that it is possible to extract useful information from this speckle pattern and to image an object buried in a random medium [1,2]. Here we present the mathematical analysis that explains the quite stunning performance of speckle imaging. Our analysis identifies a scaling regime where these schemes work well. This regime is the white-noise paraxial regime, which leads to the Itô-Schrödinger model for the wave amplitude. Our results conform with the sophisticated physical intuition that has motivated these schemes, but give a more detailed characterization of the performance. The analysis gives a description of (i) the information that can be extracted and with what resolution (ii) the statistical stability or signal-to-noise ratio with which the information can be extracted.

References

[1] J. Bertolotti, E. G. van Putten, C. Blum, A. Lagendijk, W. L. Vos, and A. P. Mosk, Non-invasive imaging through opaque scattering layers, Nature **491** (2012), pp. 232–234.

[2] J. A. Newman and K. J. Webb, Imaging optical fields through heavily scattering media, Phys. Rev. Lett. **113** (2014), 263903.

Minisymposium 2 **Thursday, 12:00-12:30, Seminar room -1.025 (20.30)**

A general framework for dynamic homogenization of wave motion at finite wavelengths and frequencies

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In this study, we establish a comprehensive framework for the homogenization of scalar wave motion in periodic media at finite frequencies and wavelegnths inside the first Brioullin zone. We take the eigenvalue problem for the unit cell of periodicity as a point of departure, and we consider the projection of germane Bloch wave function onto a suitable eigenfunction as descriptor of effective wave motion. For generality the finite wavenumber, finite frequency (FW-FF) homogenization is pursued in $\mathbb{R}^d$ via second-order asymptotic expansion about the apexes of "quadrants" comprising the first Brioullin zone, at frequencies near given (acoustic or optical) dispersion branch. We also consider degenerate situations of crossing or merging dispersion branches – relevant to the phenomenon of topological insulation, where the effective description of wave motion turns out to critically depend on the parity of a repeated eigenvalue at a given apex of the Brioullin "quadrant". For all cases considered, the effective description turns out to admit the same general framework, with differences being limited to (i) the basis eigenfunction, (ii) the reference cell of medium periodicity, and (iii) the wavenumber-frequency scaling law underpinning the asymptotic expansion.

Minisymposium 2	Thursday, 12:30-13:00, Seminar room -1.025 (20.30)

Reconstruction of local perturbations in periodic surfaces

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This talk concerns the inverse scattering problem to reconstruct a local perturbation in a periodic structure. Unlike the periodic problems, the periodicity for the scattered field no longer holds, thus classical methods, which reduce quasi-periodic fields in one periodic cell, are no longer available. Based on the Floquet–Bloch transform, a numerical method has been developed to solve the direct problem, that leads to a possibility to design an algorithm for the inverse problem. The numerical method introduced in this paper contains two steps. The first step is initialization, that is to locate the support of the perturbation by a simple method. This step reduces the inverse problem in an infinite domain into one periodic cell. The second step is to apply the Newton-CG method to solve the associated optimization problem. The perturbation is then approximated by a finite spline basis. Numerical examples are given at the end of this talk, showing the efficiency of the numerical method.

Minisymposium 2	Thursday, 15:00-15:30, Seminar room -1.025 (20.30)

Sub-wavelength sensing of bi-periodic materials using topological derivatives of the second-order homogenized moduli

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We consider scalar waves in periodic media through the lens of a second-order effective i.e. macroscopic description, aiming to compute the sensitivities of the relevant effective parameters due to topological perturbations of a microscopic unit cell. Specifically, we focus on the leading- and second-order tensorial coefficients in the governing mean-field equation. Our main result consists in the derivation and justification of the topological expansions of the homogenized moduli, and in particular of their topological derivatives. The latter are found to be computable in terms of (i) three unit-cell solutions, (ii) two adjoint solutions, and (iii) the usual polarization tensor appearing in related studies involving non-periodic media. This result may be useful toward the design of periodic media to manipulate macroscopic waves via microstructure-generated dispersion and anisotropy effects, or the sub-wavelength sensing of periodic defects or perturbations. Numerical simulations illustrate the latter possibility, where the information on long-wavelength (anisotropic) dispersion can be used to localize defects inside the unit cell.

References

[1] Bonnet M., Cornaggia R., Guzina B. B., Microstructural topological sensitivities of the second-order macroscopic model for waves in periodic media. Preprint (2018), https://hal.archives-ouvertes.fr/hal-01742396.

Minisymposium 2 **Thursday, 15:30-16:00, Seminar room -1.025 (20.30)**

Inverse Problems in Linear Elasticity via Eshelby's Integrodifferential Equation

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We present applications of the integrodifferential volume equation [1-3] corresponding to Eshelby's equivalent inclusion method for inclusions in static and dynamic linear elasticity to inverse problems for few incident fields. For the static case we use an efficient equivalent form of the equation proposed by M. Bonnet in [2] where the integrodifferential operator has norm less than one, that is, Neumann series can be used to determine the strain field. Using a first order approximation and few incident loadings and measurements we can compute numerically the material coefficients of an inclusion. For the dynamic case [2] we use the equation to determine the shape of inclusions using a Newton type scheme.

References

[1] M. Bonnet, *A modified volume integral equation for anisotropic elastic or conducting inhomogeneities. Unconditional solvability by Neumann series*, Journal of Integral Equations and Applications, Rocky Mountain Mathematics Consortium, **29** (2017), pp.271-295.

[2] M. Bonnet, *Solvability of a Volume Integral Equation Formulation for Anisotropic Elastodynamic Scattering*, Journal of Integral Equations and Applications, Rocky Mountain Mathematics Consortium, **28** (2016), pp.169-203.

[3] D. Gintides and K. Kiriaki, *Solvability of the integrodifferential equation of Eshelby's equivalent inclusion method*, The Quarterly Journal of Mechanics and Applied Mathematics, **68(1)** , (2015), pp. 85-96.

Minisymposium 3	Thursday, 11:30-12:00, Seminar room 0.014 (20.30)

On wave maps and related problems

Timothy Candy[1], **Sebastian Herr**[1,*]
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We will revisit Tataru's approach to wave maps and related equations in low regularity spaces. The aim is to find an approach via bilinear Fourier restriction estimates and atomic function spaces.

Minisymposium 3	Thursday, 12:00-12:30, Seminar room 0.014 (20.30)

Invariant measures for the periodic derivative nonlinear Schrödinger equation

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In this talk we will consider the periodic (one dimensional) derivative nonlinear Schrödinger equation with small L^2 data. We will construct an infinite sequence of invariant measures associated to the integrals of motion of the equation.

Minisymposium 3 **Thursday, 12:30-13:00, Seminar room 0.014 (20.30)**

Unique Continuation for the Z-K dispersive equation

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In this talk we will analyze uniqueness properties of solutions of the Zakharov-Kuznetsov (Z-K) equation

$$\partial_t u + \partial_x^3 u + \partial_x \partial_y^2 u + u\partial_x u = 0, \qquad (x,y) \in \mathbb{R}^2, \quad t \in [0,1]. \tag{1}$$

Mainly motivated by the very well known PDE's counterpart of the Hardy uncertainty principle, we will provide a two times unique continuation result for Z-K, which improves the result in [1]. More precisely we will prove that if the difference $u_1 - u_2$ of two solutions u_1, u_2 of (1) at two different times $t_0 = 0$ and $t_1 = 1$ decays fast enough, then $u_1 \equiv u_2$.

Moreover, as we will see, the decay rate needed in order to obtain the stated uniqueness reflects the asymptotic behavior of the fundamental solution of the associated linear problem. Encouraged by this fact we also prove the optimality of the result.

The talk is based on our recent work [2].

References

[1] E. Bustamante, P. Isaza and J. Mejía, On uniqueness properties of solutions of the Zakharov-Kuznetsov equation, *Journal of Functional Analysis* **264** (2013), pp. 2529-2549.

[2] L. Cossetti, L. Fanelli and F. Linares, Uniqueness results for Zakharov-Kuznetsov equations, (in preparation).

Minisymposium 3 **Thursday, 15:00-15:30, Seminar room 0.014 (20.30)**

Global existence and scattering via bilinear restriction estimates

Timothy Candy[1,*], **Sebastian Herr**[1]
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Bilinear restriction estimates for free waves give an efficient method to exploit both dispersion and transversality. Recently it has been shown that robust forms of these bilinear estimates also hold in certain adapted function spaces [1, 2]. We review these estimates, and explain how they can be applied to deduce small data scattering results for wave and Dirac type equations.

References

[1] T. Candy, Multiscale Bilinear Restriction Estimates for General Phases, `arXiv:1707.08944`.

[2] T. Candy and S. Herr, Transference of Bilinear Restriction Estimates to Quadratic Variation Norms and the Dirac-Klein-Gordon System, *To appear in Analysis and PDE* (2018).

Minisymposium 3 **Thursday, 15:30-16:00, Seminar room 0.014 (20.30)**

Non-selfadjoint spectral problems arising in the stability analysis of self-similar blowup in nonlinear wave equations

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Finite time break-down of solutions to nonlinear physical systems is one of the central problems in partial differential equations theory today. For equations of the so-called supercritical type it is widely believed that large initial data lead to formation of singularities in finite time. Furthermore, numerical simulations in supercritical hyperbolic systems like wave maps and Yang-Mills equations indicate that the generic blowup profile is self-similar. In fact, most of the rigorous results on the singularity formation in these models are related to the existence of self-similar solutions.

We inspect the stability of self-similar solutions and show how the analysis reduces to studying the spectrum of certain highly non-self-adjoint operators. More precisely, stability requires proving that all of the "genuine" eigenvalues of the aforementioned operators are contained in the left complex half-plane. This property goes under the name of mode stability of the underlying self-similar solution. The analysis furthermore leads to a connection problem for second order ordinary differential equations with four regular singularities. Time permitting, we will illustrate a new, general and efficient method of solving such connection problems for the purpose of proving mode stability.

Minisymposium 4 Thursday, 15:00-15:30, Seminar room 3.069 (20.30)

Shape optimization of microlenses

Alberto Paganini[1,*], **Sahar Sargheini**[2], **Ralf Hitpmair**[2], **Christian Hafner**[3]
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Microlenses are highly attractive for optical applications such as super resolution and photonic nanojets, but optimizing their design is demanding because resonance effects play an important role. In this talk, we describe a shape optimization algorithm for finite element discretization of the wave equation in frequency domain, and investigate its performance by optimizing microlenses for any reasonable value of the refraction index.

References

[1] A. Paganini, S. Sargheini, R. Hiptmair, and Ch. Hafner, Shape optimization of microlenses, *Opt. Express* **23** (2015), pp. 13099-13107.

Minisymposium 4 Thursday, 15:30-16:00, Seminar room 3.069 (20.30)

Isogeometric shape optimization for nonlinear ultrasound focusing

Markus Muhr[1], **Vanja Nikolić**[1,*], **Barbara Wohlmuth**[1], **Linus Wunderlich**[1]
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In this talk, we will discuss how focusing of high-intensity ultrasound can be improved by modifying the geometry of focusing devices through shape optimization. Our research is motivated by the need to efficiently focus ultrasound with high-power sources in, for instance, medical treatments of kidney stones and certain types of cancer.

We formulate the shape optimization problem by introducing a tracking-type cost functional to match the desired pressure distribution in the focal region [1]. Westervelt's equation, a quasilinear acoustic wave equation, is used as a model of the pressure field. We employ the optimize first, then discretize approach and first rigorously compute the shape derivative of our cost functional. A gradient-based optimization algorithm is then developed within the concept of isogeometric analysis, where the geometry is exactly represented by splines at every optimization step and the same basis is used to approximate the equations. Numerical experiments in a 2D setting will illustrate our findings.

References

[1] M. Muhr, V. Nikolić, Barbara Wohlmuth and Linus Wunderlich, Isogeometric shape optimization for nonlinear ultrasound focusing, *Evolution Equations & Control Theory*, to appear.

Minisymposium 4 **Thursday, 16:30-17:00, Seminar room 3.069 (20.30)**

Pareto optimization of resonances

Illya M. Karabash[1,*]
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Resonators with small decay rate are required in Optical Engineering because they enhance intrinsically small light-matter interactions. Since light is difficult to localize, it is hard to realize small-sized cavities with strong light confinement (high-Q resonators). Mathematically, the problem is to design, under certain constraints, a structure of the medium in such a way that it generates a resonance (or quasi-normal-eigenvalue) as close as possible to the real line. While engineering and computational aspects of fabrication of high-Q resonators have drawn great attention during the last decade, the analytic background of spectral optimization for non-Hermitian eigenproblems is still in the stage of development.

The goal of the talk is to present a rigorous approach to resonance optimization. The fact that resonances move in the complex plane brings to the problem the features of Pareto optimization and makes it necessary to develop a multi-parameter approach to the perturbation theory of resonances.

Minisymposium 4 **Thursday, 17:00-17:30, Seminar room 3.069 (20.30)**

Fast estimation of losses of free-form waveguides using a fundamental mode approximation

Fernando Negredo[1,*], **Matthias Blaicher**[1], **Aleksandar Nesic**[1], **Pascal Kraft**[1], **Julian Ott**[1], **Willy Dörfler**[1], **Christian Koos**[1], **Carsten Rockstuhl**[1]
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Free-form waveguides, also called potonic wire bonds (PWB), are an essential part of a future generation of integrated photonic multi-chip modules. They serve the purpose to link individual sub-components that are made from different materials. In the fabrication process of these multi-chip modules, we face the challenge to identify the best trajectory, i.e. the trajectory that suffers from the least losses, of the PWB that connects a given input and output port of some modules in pretty short time; ideally within seconds. This denies the use of full wave numerical solvers for this purpose and prompts instead for the development of suitable approximate tools that are fast and predictive.

Here, we rely on the fundamental mode approximation to reliably predict the losses of a given PWB. The method divides the trajectory into discrete sections of waveguides with a constant radius of curvature. In each of these sections we consider only the fundamental mode, i.e. the mode with the least losses. This is justified, as all other modes have a much larger imaginary part. The total losses are then calculated from, (a) the bending losses in the different sections that occur due to the finite radius of curvature and (b) the transition losses that occur at the interface between waveguide sections of different geometry due to a modal mismatch. The method relies on a pre-calculation of those modal properties and enables the optimisation of the trajectory of PWBs within the desired time scale.

Minisymposium 5 **Tuesday, 15:00-15:30, Seminar room 1.067 (20.30)**

Amplitude equations for spatially periodic water wave models

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We present a number of results about the validity of amplitude equations for periodic water wave models. We consider the KdV approximation for a Boussinesq equation with spatially periodic coefficients and the NLS approximation for a system of coupled nonlinear wave equations which exhibits a spectral picture similar to the spatially periodic water wave problem.

Minisymposium 5 **Tuesday, 15:30-16:00, Seminar room 1.067 (20.30)**

Regularity of the highest wave for the reduced Ostrovsky equation

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We discuss periodic traveling wave solutions of the reduced Ostrovsky equation, which arises in the context of long surface and internal gravity waves in a rotating fluid. Our aim is to reformulate this local equation into a nonlocal dispersive equation, and study the properties of their traveling waves by analyzing the corresponding convolution kernel. Of particular interest is the existence of a highest, cusped, traveling wave solution, which we obtain as a limiting case at the end of a global bifurcation branch. We show that the regularity at the crest of a highest wave is precisely Lipschitz. While the reduced Ostrovsky equation is of order -2, similar investigations have been done for equations of different orders. This work is part of a longer study with the aim of understanding the interaction of nonlinearities and dispersion, which lead to families of ever-higher waves, ending in a typically singular and highest one.

This is a joint work with M. Ehrström and R. N. Dhara.

Minisymposium 5 Wednesday, 11:30-12:00, Seminar room -1.025 (20.30)

Modified Energy Functionals, Normal Forms and the NLS Approximation

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We consider a model equation which captures many important aspects of the water wave problem and show that by combining normal forms with a modified energy functional one can accurately approximate wave packet solutions of this equation by solutions of the Nonlinear Schroedinger equation in Sobolev spaces of finite regularity.

Minisymposium 5 Wednesday, 12:00-12:30, Seminar room -1.025 (20.30)

Validity of the Nonlinear Schrödinger approximation for quasilinear dispersive equations

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We consider a nonlinear dispersive equations with a quasilinear quadratic term. We derive the Nonlinear Schrödinger (NLS) equation as a formal approximation equation describing the evolution of the envelopes of oscillating wave packet-like solutions to the quasilinear dispersive equations, and justify the NLS approximation with the help of error estimates between exact solutions to the quasilinear dispersive equations and their formal approximations obtained via the NLS equation.
The proof relies on estimates of an appropriate energy whose construction is inspired by the method of normal-form transforms. We have to overcome difficulties caused by the occurrence of resonances and avoid a loss of regularity resulting from the quasilinear term.
Some ideas of this method of proof can be used to justify the NLS approximation for the arc length formulation of the two-dimensional water wave problem in case of finite depth of water and with and without surface tension.
This is a joint work with Wolf-Patrick Düll.

Minisymposium 5 **Wednesday, 12:30-13:00, Seminar room -1.025 (20.30)**

Spatial asymptotics for solitary waves in deep water

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We consider the behavior near spatial infinity of a localized traveling wave on the surface of an infinitely deep fluid. In a variety of settings and under suitable decay assumptions, we show that the leading order term in these asymptotics is of dipole type. This has many implications for the wave, particularly in the simpler settings where the dipole moment in the expansion is given explicitly in terms of the kinetic energy. As an application, we provide detailed asymptotics for the waves with compactly supported vorticity constructed in [1].

References

[1] J. Shatah, S. Walsh, and C. Zeng, Travelling water waves with compactly supported vorticity, Nonlinearity **26** (2013), pp. 1529–1564.

Minisymposium 5 **Wednesday, 15:00-15:30, Seminar room -1.025 (20.30)**

Degenerate dispersive equations

Pierre Germain[1], **Benjamin Harrop-Griffiths**[1,*], **Jeremy L. Marzuola**[2]

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Degenerate dispersive equations, where the dispersive effects degenerate at a point in physical space, appear in several physical contexts, including shallow water waves. In this talk we discuss some recent work on toy models for this phenomenom.

Minisymposium 5 Wednesday, 15:30-16:00, Seminar room -1.025 (20.30)

A Morawetz inequality for water waves

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We consider gravity and gravity/capillary water waves in two space dimensions. Assuming uniform energy bounds for the solutions, we prove local energy decay estimates. Our result is uniform in the infinite depth limit.

Minisymposium 5 Wednesday, 16:30-17:00, Seminar room -1.025 (20.30)

Multi-modal and non-symmetric steady water waves with vorticity

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We consider the nonlinear problem of steady gravity-driven waves on the free surface of a two-dimensional flow of an incompressible fluid. We neglect the surface tension and assume a flow to be rotational and also allow interior stagnation points. In the talk we will discuss a recent progress in the theory of small-amplitude steady water waves with vorticity. Such waves are small perturbations of uniform streams with a several counter-currents. In contrast to the irrotational case of zero vorticity, when only Stokes and Solitary waves exist, the class of small-amplitude waves with vorticity includes multi-modal and non-symmetric waves and, possibly, waves with even more complicated geometry.

Minisymposium 5 **Wednesday, 17:00-17:30, Seminar room -1.025 (20.30)**

Wave collapses and turbulence at the free surface of a liquid dielectric in an external tangential electric field

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The wave turbulence is observed at interaction of nonlinear dispersive waves in many physical processes. In recent works [1,2], it was found that under certain conditions, regions with a high energy concentration can form at the liquid boundary under the action of strong external tangential electric field. In these regions electrostatic and dynamic pressures undergo a discontinuity. The formation of such discontinuities triggers a direct energy cascade, which leads to the transfer of energy from large scales to small ones. We numerically demonstrate that this process can lead to the development of surface wave turbulence.

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[1] E.A. Kochurin *J. Apl. Mech, Tech. Phys.* **59** (1) (2018), pp. 79–85.

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Minisymposium 5 **Wednesday, 17:30-18:00, Seminar room -1.025 (20.30)**

Propagation of long-crested water waves

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In this talk we will present some results about the propagation of waves such as those sometimes observed in canals and in near-shore zones of large bodies of water [1]. A special interest will also be on waves arising in bore propagation [2], when a surge of water invades an otherwise constantly flowing river. The results are developed in the context of Boussinesq models, so they are applicable to waves that have small amplitude and long wave length when compared with the undisturbed depth. We will discuss the theory of well-posednses results on the long, Boussinesq time scale. In the case of bore propagation, where the mass of water has an infinite energy a priori, we will show how to use suitable approximations with which to compare the full solution.

References

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Minisymposium 6 — Monday, 11:45-12:15, Seminar room 0.014 (20.30)

The Generalized Linear Sampling Method for a Far-Field Inverse Scattering Problem in the Time Domain

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In Memory of Armin Lechleiter

Dear Friend and Colleague, An Extraordinary Mathematician!

We develop the generalized linear sampling method for solving the inverse scattering problem for the wave equation using time dependent far field measurements. This method provides a mathematically rigorous characterization of the support of the scatterer, and is also justifed for noisy data. The use of time domain measurements is a remedy for large amount of spatial data typically needed for the application qualitative reconstruction methods in inverse scattering theory.

Minisymposium 6 — Monday, 12:15-12:45, Seminar room 0.014 (20.30)

Multifrequency MUSIC and a multifrequency factorization method for inverse scattering problems

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We consider inverse scattering problems for time-harmonic waves governed by the Helmholtz equation. We aim to recover information about the position and the geometry of several penetrable scattering objects in an otherwise homogeneous background medium using far field measurements. We assume that the far field data is available for a whole frequency band but only few observation directions.

In the first part we present a MUSIC-type reconstruction method for recovering the positions of several small scattering objects based on an asymptotic expansion of the far field pattern with respect to the size of the inhomogeneities. We provide rigorous theoretical bounds on the number of wavenumbers and observation directions necessary to reconstruct a certain number of scatterers.

In the second part we turn our attention to inverse scattering problems in Born approximation. We present a factorization-type method involving an appropriate far field operator that uses multifrequency far field data. The method produces a union of convex polygons with normals in the observation directions that approximate the positions and the geometry of well-separated scatterers.

Minisymposium 6 Monday, 12:45-13:15, Seminar room 0.014 (20.30)

Inverse Problems for Perturbed Bi-harmonic Operator

Valery Serov[1,*]
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This work deal with the inverse spectral and inverse scattering problems for the perturbed bi-harmonic operator with singular coefficients.

The first type of inverse problems are: (1) do the Dirichlet-to-Neumann map uniquely determine the singular coefficients of the operator? (2) do the Dirichlet eigenvalues and the derivatives of the normalized eigenfunctions at the boundary of this operator (Borg-Levinson data) uniquely determine its coefficients? The first result is: the knowledge of the Dirichlet eigenvalues and the derivatives of the normalized eigenfunctions at the boundary up to order three uniquely determine the Dirichlet-to-Neumann map corresponding to this operator. And the second result is: the knowledge of the Dirichlet-to-Neumann map uniquely determine the coefficients of the given perturbation of order two for the bi-harmonic operator.

The second type of inverse problems is the classical scattering theory. The first result here is the analogue of Saito's formula which states also the uniqueness in the inverse scattering problems with full data. Another consideration concerns to the Born approximation for backscattering problem. And the main result here is: the inverse backscattering Born approximation allows us to reconstruct the singularities of unknown coefficients of the perturbed bi-harmonic operator (based on the backscattering amplitude). One more result shows that the knowledge of the Green's function (corresponding to this perturbation) for all $x, y \in R^3$ and for the spectral parameter $k \to \infty$ uniquely determines the unknown potential. Moreover, in this case we can calculate effectively the Fourier transform of it.

Minisymposium 6 Monday, 15:00:-15:30, Seminar room 0.014 (20.30)

Eigenvalue Problems in Inverse Scattering Theory

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The material properties of an anisotropic media are in general not uniquely determined by the measured scattering data, thus creating obvious difficulties in the nondestructive testing of such materials. One way to overcome this difficulty is to use "target signatures" that can be determined from the scattering data, i.e. eigenvalues that are associated with the scattering operator. Examples of this approach are the theory of scattering resonances and the theory of transmission eigenvalues. However, the use of these target signatures is problematic since the scattering resonances are complex and transmission eigenvalues are real only for materials that have no absorption. In this talk, through the use of modified far field operators, we will introduce new classes of eigenvalue problems that overcome the problems associated with scattering resonances and transmission eigenvalues.

Minisymposium 6	Monday, 15:30-16:00, Seminar room 0.014 (20.30)

The Monotonicity Method for the Helmholtz equation

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This talk presents results from the recent work [2]. We extend monotonicity-based methods [3] in inverse problems to the case of the Helmholtz (or stationary Schrödinger) equation $(\Delta + k^2 q)u = 0$ in a bounded domain for fixed non-resonance frequency $k > 0$ and real-valued scattering coefficient function q. We show a monotonicity relation between the scattering coefficient q and the local Neumann-Dirichlet operator that holds up to finitely many eigenvalues. Combining this with the method of localized potentials [1], or Runge approximation, adapted to the case where finitely many constraints are present, we derive a constructive monotonicity-based characterization of scatterers from partial boundary data. We also obtain the local uniqueness result that two coefficient functions q_1 and q_2 can be distinguished by partial boundary data if there is a neighborhood of the boundary part where $q_1 \geq q_2$ and $q_1 \not\equiv q_2$.

References

[1] Bastian Gebauer, Localized potentials in electrical impedance tomography, *Inverse Probl. Imaging* **2** (2008), pp. 251–269.

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Minisymposium 6	Monday, 16:30-17:00, Seminar room 0.014 (20.30)

The imaginary part of the scattering Green function: monochromatic relations to the real part and uniqueness results for inverse problems

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For many wave propagation problems with random sources it has been demonstrated that cross correlations of wave fields are proportional to the imaginary part of the Green function of the underlying wave equation. This leads to the inverse problem to recover coefficients of a wave equation from the imaginary part of the Green function on some measurement manifold. In this talk I will present, in particular, local uniqueness results for the Schrödinger equation with one frequency and for the acoustic wave equation with unknown density and sound speed and two frequencies. As the main tool of the analysis, I will present new algebraic identities between the real and the imaginary part of Green's function, which in contrast to the well-known Kramers-Kronig relations involve only one frequency.

Minisymposium 6 **Monday, 17:00-17:30, Seminar room 0.014 (20.30)**

Logarithmic linearization in electrical impedance tomography

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The aim of electrical impedance tomography is to reconstruct the electric conductivity inside a physical body from surface measurements of currents and voltages. As the boundary current-voltage pairs depend nonlinearly on the conductivity, impedance tomography leads to a nonlinear inverse problem. The forward problem of impedance tomography is often linearized with respect to the conductivity and the resulting linear inverse problem is regarded as a subproblem in an iterative algorithm or as a simple reconstruction method as such. We compare this standard linearization approach to linearizations with respect to the resistivity or the logarithm of the conductivity and numerically demonstrate the conductivity linearization usually leads to compromised accuracy. Inspired by these observations, we present and analyze a new linearization technique that is based on the logarithm of the Neumann-to-Dirichlet boundary map. The introduced method is directly applicable to discrete settings, including the complete electrode model. Numerical examples indicate that the proposed method is an accurate way of linearizing the problem of electrical impedance tomography. For more information, see [1].

References

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Minisymposium 6 **Monday, 17:30-18:00, Seminar room 0.014 (20.30)**

A Single Boundary Integral Equation for Transmission Eigenvalues

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We present a single boundary integral equation for the transmission eigenvalue for a medium with constant refractive index that is based on the Robin-to-Neumann operator for a non-local impedance boundary value problem. Its use for the numerical computation of transmission eigenvalues via Beyn's algorithm for non-linear eigenvalue problems for analytic operators is illustrated.

References

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Minisymposium 7 **Monday, 11:45-12:15, Seminar room 1.067 (20.30)**

Numerical analysis of a structure preserving scheme for Maxwell Klein-Gordon equations in 2D

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In this work, we focus on a special class of Hamiltonian nonlinear wave equations from which Maxwell Klein-Gordon equations serve as a special example. These models have a gauge invariance property that imposes an additional constraint equation.
We propose to develop the numerical analysis of leap-frog type time integration schemes that preserve this structure at the discrete level. We therefore require a discrete gauge invariance of both the kinetic and potential part of a discrete Hamiltonian. The strategy of convergence proof then relies on stability estimates that are obtained, under a CFL condition, via a discrete energy principle and a boot-strap type argument. Compactness arguments complete the proof and lead to convergence of the scheme. We illustrate the possibility of such gauge invariant schemes with the special case of Maxwell Klein-Gordon equations in 2D. We use a spatial discretization relying on a Finite Element framework and we recover the discrete gauge invariance thanks to the use of Lattice-Gauge theory.

Minisymposium 7 **Monday, 12:15-12:45, Seminar room 1.067 (20.30)**

Stable and convergent fully discrete interior–exterior coupling for Maxwell's equations and related problems

Balázs Kovács[1,*], **Christian Lubich**[1]
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Maxwell's equations are considered with transparent boundary conditions, for initial conditions and inhomogeneity having support in a bounded, not necessarily convex three-dimensional domain or in a collection of such domains.

We also give some outlook on acoustic and elastic wave equations, and on problems with further models coupled to Maxwell's equations.

The numerical method only involves the interior domain and its boundary. The transparent boundary conditions are imposed via a time-dependent boundary integral operator that is shown to satisfy a coercivity property. The stability of the numerical method relies on this coercivity and on an anti-symmetric structure of the discretized equations that is inherited from a weak first-order formulation of the continuous equations. For Maxwell's equations the method proposed here uses a discontinuous Galerkin method and the leapfrog scheme in the interior and is coupled to boundary elements and convolution quadrature on the boundary. The method is explicit in the interior and implicit on the boundary. Stability and convergence of the spatial semidiscretization are proven, and with a computationally simple stabilization term, this is also shown for the full discretization.

References

[1] B. Kovács and Ch. Lubich. Stable and convergent fully discrete interior–exterior coupling of Maxwell's equations. *Numerische Mathematik*, 137(1):91–117, 2017.

Minisymposium 7 **Monday, 12:45-13:15, Seminar room 1.067 (20.30)**

Error analysis of an ADI splitting for discontinuous Galerkin discretizations of linear wave-type problems

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In this talk we investigate the convergence of an alternating direction implicit (ADI) method for discontinuous Galerkin discretizations of wave-type equations on product domains. This method is both unconditionally stable and computationally cheap (each time step is of linear complexity).

We prove that the scheme converges with optimal order in time and space with constants which are independent of the spatial mesh widths and thus are robust under mesh refinements. The proof is based on techniques established for the full discretization of a locally implicit scheme comprising the Crank–Nicolson and the Verlet scheme in [1]. In fact, we show that similar to the Verlet scheme, the ADI scheme under consideration can be interpreted and analyzed as a perturbation of the Crank–Nicolson scheme.

References

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Minisymposium 7 **Monday, 15:00:-15:30, Seminar room 1.067 (20.30)**

Convergence analysis of conservative local time discretization for wave equations

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In this talk we present and analyze a time discretization strategy for linear wave propagation that aims at using locally in space the most adapted time discretization among a family of implicit or explicit centered second order schemes. The domain of interest being decomposed into several regions, different time discretization can be chosen depending on the local properties of the spatial discretization (mesh size and quality, order of the finite elements) or the physical parameters (high wave speed, low density). We show that, under some conditions on the time step, the family of time discretization obtained combined with standard high order finite element method ensures second order convergence in time and space.

Minisymposium 7 **Monday, 15:30-16:00, Seminar room 1.067 (20.30)**

On leap-frog-Chebyshev methods

Marlis Hochbruck[1], **Andreas Sturm**[1,*]
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In this talk we consider wave-type problems of the form

$$u'' = -Lu + g(u),$$

where L is a symmetric, positive definite matrix stemming e.g. from the spatial discretization of the Laplacian, and g a is a "nice" non-linearity, which is expensive to evaluate. The most popular integrator for such problems is the leap-frog scheme

$$u^{n+1} - 2u^n + u^{n-1} = -\tau^2 L u^n + \tau^2 g(u^n),$$

which is stable under the CFL condition $\tau^2 \|L\| < 4$. Typically the norm of L is very large which forces the use of a tiny time step size τ. Thus, the scheme requires numerous evaluations of the non-linearity g. In order to overcome this issue we propose to modify the leap-frog scheme to

$$u^{n+1} - 2u^n + u^{n-1} = -P_p(\tau^2 L)u^n + \tau^2 g(u^n).$$

Here, P_p is a scaled and translated Chebyshev polynomial of degree p and thus we term this scheme leap-frog-Chebyshev method. In our talk we present a quite general stability result for such schemes. Moreover, we show that it is second order convergent similar to the leap-frog scheme but allows a time step size which is (almost) p times larger. Numerical examples show the improved properties and the higher efficiency of this novel scheme.

Minisymposium 7 **Monday, 16:30-17:00, Seminar room 1.067 (20.30)**

Numerical methods and analysis for the Zakharov system in the subsonic limit regime

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We present two uniformly accurate numerical methods for discretizing the Zakharov system (ZS) with a parameter $0 < \varepsilon \leq 1$, which is inversely proportional to the acoustic speed. In the subsonic limit regime, i.e., $0 < \varepsilon \ll 1$, the solution of the ZS propagates waves with $O(\varepsilon)$-wavelength in time due to the singular perturbation of the wave operator in ZS and/or the incompatibility of the initial data. A uniformly accurate finite difference method is proposed and analyzed based on an asymptotic consistent formulation. The method is proven to be convergent quadratically and linearly in space and time, respectively, which is uniformly for ε. By introducing a multiscale decomposition of ZS, we propose a time-splitting multiscale time integrator (TS-MTI) method, which is uniformly and optimally accurate in both space and time with spectral and quadratic convergence rate, respectively. Finally, the TS-MTI method is applied to study numerically convergence rates of ZS to its limiting models when $\varepsilon \to 0^+$.

References

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Minisymposium 7 **Monday, 17:00-17:30, Seminar room 1.067 (20.30)**

Stability and convergence of time discretizations of quasi-linear evolution equations of Kato type

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Semidiscretization in time is studied for a class of quasi-linear evolution equations in a framework due to Kato, which applies to symmetric first-order hyperbolic systems and to a variety of fluid and wave equations. In the regime where the solution is sufficiently regular, we show stability and optimal-order convergence of the linearly implicit and fully implicit midpoint rules and of higher-order implicit Runge–Kutta methods that are algebraically stable and coercive, such as the collocation methods at Gauss nodes.

We show that Kato's original framework from 1975 combines remarkably well with the technique of "energy estimates" for time discretizations, that is, with the use of positive definite and semi-definite bilinear forms for proving stability and error bounds. We show this for implicit Runge–Kutta methods such as the Gauss and Radau IIA methods of arbitrary orders, which have the properties of algebraic stability and coercivity, notions that are due to Burrage and Butcher and to Crouzeix (for algebraic stability) and to Crouzeix and Raviart (for coercivity). Although these notions were developed and recognized as important properties in the context of stiff ordinary differential equations in the same decade in which Kato's paper appeared, it seems that no link between these analytical and numerical theories was made. With a delay of some decades, this is done in the present talk - in view of both, the perfectly fitting connection of the analytical framework and the numerical methods, and the undiminished significance of the considered evolution equations in applications.

Minisymposium 7 **Monday, 17:30-18:00, Seminar room 1.067 (20.30)**

On the convergence of Lawson methods for semilinear stiff problems

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Since their introduction in 1967, Lawson methods have attracted continuous interest for the time discretization of evolution equations. The popularity of these methods is in some contrast to the fact that they may have a bad convergence behaviour, since they do not satisfy any of the stiff order conditions. The aim of this talk is to explain this discrepancy. It is shown that non-stiff order conditions together with appropriate regularity assumptions imply high-order convergence of Lawson methods.

The precise regularity assumptions required for high-order convergence are worked out in this talk and related to the corresponding assumptions for splitting schemes. In contrast to previous work the analysis is based on expansions of the exact and the numerical solution along the flow of the homogeneous problem. Numerical examples for the Schrödinger equation are included.

Minisymposium 8 **Wednesday, 11:30-12:00, Seminar room 2.067 (20.30)**

The Maslov index in symplectic Banach spaces

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I shall summarize the main results and highlights of [1], where we consider a curve of Fredholm pairs of Lagrangian subspaces in a fixed Banach space with continuously varying weak symplectic structures. Assuming vanishing index of the Fredholm pairs, we obtain intrinsically a continuously varying splitting of the total Banach space into pairs of symplectic subspaces. Using such decompositions we define the Maslov index of the curve by symplectic reduction to the classical finite-dimensional case. We prove the transitivity of repeated symplectic reductions and obtain the invariance of the Maslov index under symplectic reduction, while recovering all the standard properties of the Maslov index.

As an application, we consider curves of elliptic operators which have varying principal symbol, varying maximal domain and are not necessarily of Dirac type. For this class of operator curves, we derive a desuspension spectral flow formula for varying well-posed boundary conditions on manifolds with boundary and obtain the splitting formula of the spectral flow on partitioned manifolds.

References

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Minisymposium 8 **Wednesday, 12:00-12:30, Seminar room 2.067 (20.30)**

Iteration theory of Maslov-type index

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In this talk, I will give a brief review of the iteration of Maslov-type index for symplectic paths developed by Yiming Long and his coauthors. Then we use it to show that there exists at least $[n/2] + 1$ closed characteristics on compact convex C^2 hypersurfaces in $\mathbb{R}^{2n}$.

References

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Minisymposium 8 **Wednesday, 12:30-13:00, Seminar room 2.067 (20.30)**

On the Fredholm Lagrangian Grassmannian, Spectral Flow and ODEs in Hilbert Spaces

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We consider homoclinic solutions for Hamiltonian systems in symplectic Hilbert spaces and generalise spectral flow formulas that were proved by Pejsachowicz and the author in finite dimensions some years ago. Roughly speaking, our main theorem relates the spectra of infinite dimensional Hamiltonian systems under homoclinic boundary conditions to intersections of their stable and unstable spaces. We make use of striking results by Abbondandolo and Majer to study Fredholm properties of infinite dimensional Hamiltonian systems.

References

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Minisymposium 8 **Thursday, 11:30-12:00, Seminar room 2.066 (20.30)**

The Maslov Index and the Spectra of Second Order Elliptic Operators

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In this talk I will discuss a formula relating the spectral flow of the one-parameter families of second order elliptic operators to the Maslov index, the topological invariant counting the signed number of conjugate points of certain paths of Lagrangian planes. In addition, I will present formulas expressing the Morse index, the number of negative eigenvalues, in terms of the Maslov index for several classes of second order differential operators.

Minisymposium 8 **Thursday, 12:00-12:30, Seminar room 2.066 (20.30)**

A generalized Maslov index for non-Hamiltonian systems

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The Maslov index is a powerful and well known tool in the study of Hamiltonian systems, providing a generalization of Sturm-Liouville theory to systems of equations. For non-Hamiltonian systems, one no longer has the symplectic structure needed to define the Maslov index. In this talk I will describe a recent construction of a "generalized Maslov index" for a very broad class of differential equations. The key observation is that the manifold of Lagrangian planes can be enlarged considerably without altering its topological structure, and in particular its fundamental group. This is joint work with Tom Baird, Paul Cornwell, Chris Jones and Robert Marangell.

Minisymposium 8 **Thursday, 12:30-13:00, Seminar room 2.066 (20.30)**

A Morse-Maslov theorem for the nonlinear Schrödinger equation on graphs

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In this talk I will discuss how the Maslov index can be used to get an estimate on the Morse index for standing waves of the nonlinear Schroödinger (NLS) equation on a quantum graph. First I will discuss how to construct a graph on which there is a standing wave solution to the NLS. Then with a solution in hand, linearising produces a familiar pair of coupled operators whose spectrum we are interested in. I will then recast the eigenvalue problem in the space of Lagrangian planes and relate the Maslov index to the spectrum of each of the operators individually. If there is time I will talk about correction factors to this computation.

Minisymposium 8 Thursday, 15:00-15:30, Seminar room 2.066 (20.30)

Index and instability of closed semi-Riemannian geodesics

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A celebrated result due to Poincaré asserts that a closed minimizing geodesic on a orientable surface is linearly unstable when considered as orbit of the co-geodesic flow. In this talk, starting from this classical theorem, we discuss some recently new results on the instability and hyperbolicity of closed (maybe not minimizing) geodesics of any causal character on higher dimensional (even not orientable) semi-Riemannian manifolds. Dropping the non-positivity assumption of the metric tensor is a quite challenging task since the Morse index is truly infinite.

References

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Minisymposium 8 Thursday, 15:30-16:00, Seminar room 2.066 (20.30)

Fluidic Shock Waves without or with Electromagnetic Fields

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The talk surveys what is known about the existence and stability of single-shock solutions to the equations of compressible fluid flow, coupled, or not, to Maxwell's equations of electromagnetism, considering or neglecting dissipative effects. While at least for standard fluids, shock waves of all strenghts seem to be stable also in several space dimensions (Majda, Zumbrun), multidimensional fluidic shocks in the presence of electromagnetic fields allow for transitions from stability to instability (Trakhinin, Freistühler et al.). One studies Kreiss-Lopatinski determinants and Evans functions.

Minisymposium 8	Thursday, 16:30-17:00, Seminar room 2.066 (20.30)

Fredholm Grassmannian flows and nonlinear PDEs

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We consider evolutionary partial differential systems with nonlocal nonlinearities. For arbitrary initial data, we demonstrate how solutions to large classes of such systems can be generated from the corresponding linearized equations. The key is a linear Fredholm integral equation relating the linearized flow to an auxiliary linear flow. The solution to this Fredholm equation is the representation in a given coordinate patch of the underlying Fredholm Grassmannian of the combined linearized and auxiliary linear flow. It is analogous to the Marchenko integral equation in integrable systems. We show explicitly how this can be achieved through several examples including reaction-diffusion systems with nonlocal quadratic nonlinearities, the nonlinear Schrödinger equation with a nonlocal cubic nonlinearity and stochastic partial differential equations with nonlocal nonlinearities. In each case we demonstrate our approach with numerical simulations. We discuss how our approach might be extended.

References

[1] Beck M, Doikou A, Malham SJA, Stylianidis I. 2018 Grassmannian flows and applications to nonlinear partial differential equations, *Proc. Abel Symposium*, submitted.

[2] Beck M, Doikou A, Malham SJA, Stylianidis I. 2018 Partial differential systems with nonlocal nonlinearities: Generation and solutions. *Phil. Trans. A* **376**: 20170195.

Minisymposium 8	Thursday, 17:00-17:30, Seminar room 2.066 (20.30)

The Maslov and Morse indices for Hamiltonian systems

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We establish the relation between the Maslov and Morse indices for general Hamiltonian systems with block matrix coefficients. In particular, we extend the results of the classical oscillation theory originally derived for the scalar Sturm–Liouville and Dirac-type operators to general Hamiltonian systems in the intervals of essential spectral gaps.

Minisymposium 8 — Thursday, 17:30-18:00, Seminar room 2.066 (20.30)

On coalescing characteristics in Whitham modulation theory: the (Krein) sign characteristic and its nonlinear implications

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The change of type of the Whitham modulation equations from elliptic to hyperbolic is signaled, in the linearization, by a coalescence of purely imaginary eigenvalues. It looks like two eigenvalues with opposite Krein signature meeting and generating a Hamiltonian Hopf bifurcation. However, there is no natural Hamiltonian structure. Instead we find that the linearization is more naturally interpreted as a Hermitian operator pencil so the invariant is the "sign characteristic" which is more general than Krein signature in that zero eigenvalues also have a sign. This theory becomes more elaborate and interesting for multiphase Whitham modulation theory and is essential for studying coalescence of characteristics. The theory is formulated, the sign defined, and then the nonlinear implications are studied. By rescaling and reformulating near coalescing characteristcs it is found that the nonlinear behaviour is governing by a universal two-way Boussinesq equation. An example is the coalescence of characteristics in the family of two-phase wavetrains in the coupled NLS equation.

References

[1] T.J. Bridges and D.J. Ratliff, On the elliptic-hyperbolic transition in Whitham modulation theory, SIAM J. Appl. Math. **77** (2017), pp. 1989–2011.

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Minisymposium 9 **Monday, 11:45-12:15, Seminar room 2.066 (20.30)**

Polarization modulational instability in microresonators

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We report a theoretical study of the generation of optical frequency combs in Kerr microresonators that feature nonlinear mode coupling of TE/TM polarization modes. The state of polarization introduces an additional degree of freedom that can lead to the appearence of new modulational instabilities as compared to the scalar case. Such instabilities are associated with the growth of different normal modes and can result in the generation of novel types of frequency comb states. Our study extends previous work on polarization modulational instability in fiber-ring cavities [1] to conditions that are valid for Kerr microresonators. The comb dynamics is modelled using two incoherently coupled temporal Lugiato-Lefever equations that include the effects of cross-phase modulation and group-velocity mismatch. A detailed investigation is made of the modulational instability, and we present simple phase-matching conditions together with numerical solutions for two complementary limiting cases.

References

[1] M. Haelterman, S. Trillo, and S. Wabnitz, "Polarization multistability and instability in a nonlinear dispersive ring cavity," *J. Opt. Soc. Am. B* **11** (1994), pp. 446–456.

Minisymposium 9 **Monday, 12:15-12:45, Seminar room 2.066 (20.30)**

Cracking patterns in optical microresonators

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The formation and stability of patterns and localized states in optical microresonators determine the properties of the frequency combs at the output of the cavity [1]. In experiments a number of different outputs are observed. Periodic patterns as a result of a Turing or modulation instability correspond to perfect combs with a separation between teeth of several free spectral ranges. However, sometimes frequency combs are spoiled by the appearance of side bands around the main peaks. In this work we show that such spectra are the result of Eckhaus instabilities of periodic patterns [2]. The resulting states, known as cracking patterns, are not perfectly periodic and consist of portions of a pattern separated by the homogeneous state, which can also be interpreted as arbitrary sequences of localized states (dissipative solitons) and holes in a pattern.

References

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Minisymposium 9 **Monday, 12:45-13:25, Seminar room 2.066 (20.30)**

Microresonator soliton frequency combs

Tobias Kippenberg[1,*], Hairun Guo[1], John Jost[1], Martin H.P. Pfeiffer[1], Erwan Lucas[1], Maxim Karpov[1], Junqiu Liu[1], Miles Anderson[1], Arslan S. Raja[1], Bahareh Ghadiani[1], Anton Lukashschuk[1], Wenle Weng[1], Romain Bouchand[1], Jia-Jung Ho[1], Michael Geiselmann[2]

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Microresonator-based frequency combs [1] provide access to compact, photonic-chip-integrated optical combs with large mode spacing. It was recently demonstrated that they can operate in the regime of dissipative Kerr solitons corresponding to the formation of stable intracavity pulses [2], which enable fully coherent and broadband optical combs spanning more than an octave [3]. We will discuss the rich physics of such dissipative Kerr solitons, including their behaviour under impact of various nonlinear, thermal and other effects induced by the microresonator, as well as different nonstationary regimes of their operation (breathing). We will also highlight several important application of such microresonator solitons in telecommunications and distance measurements.

References

[1] T. J. Kippenberg, R. Holzwarth and S. A. Diddams , Microresonator-Based Optical Frequency Combs, *Science* **332** (2011), pp. 555–559.

[2] T. Herr *et al.*, Temporal solitons in optical microresonators, in *Nature Photonics* **8** (2014) pp. 145–152.

[3] M. H. P. Pfeiffer *et al.*, Octave-spanning dissipative Kerr soliton frequency combs in Si_3N_4 microresonators, *Optica* **4(7)** (2017) pp. 684–691.

Minisymposium 9 **Monday, 15:00:-15:30, Seminar room 2.066 (20.30)**

Periodic waves of the Lugiato-Lefever equation at the onset of Turing instability

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The Lugiato-Lefever equation is one of the mathematical models used for the description of optical Kerr frequency combs. Starting from a detailed description of the stability properties of constant solutions, we then focus on the periodic steady waves which bifurcate at the onset of Turing instability. Using a center manifold reduction, we analyze these Turing bifurcations, and prove the existence of periodic steady waves. This approach also allows us to conclude on the nonlinear orbital stability of these waves for co-periodic perturbations, i.e., for periodic perturbations which have the same period as the wave. This stability result is completed by a spectral stability analysis for general bounded perturbations.

Minisymposium 9 **Monday, 15:30-16:10, Seminar room 2.066 (20.30)**

Instabilities of periodic waves for the Lugiato-Lefever equation

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The Lugiato-Lefever equation is one of the mathematical models used for the description of optical Kerr frequency combs. Relying upon reduction methods from bifurcation theory, we study the existence and the stability of periodic steady waves generated by fully developed instabilities of constant solutions. For the stability question, we distinguish between shape instabilities, which are due to the shape of the periodic wave itself, and background instabilities, which are inherited from the instabilities of the constant background solution.

Minisymposium 9 **Monday, 16:30-17:00, Seminar room 2.066 (20.30)**

Bifurcation structure of localized states in the Lugiato-Lefever equation with anomalous dispersion

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The origin, stability and bifurcation structure of different types of bright localized structures described by the Lugiato-Lefever equation is studied. This mean field model describes the nonlinear dynamics of light circulating in fiber cavities and microresonators. In the case of anomalous group velocity dispersion, we show that for low values of the intracavity phase detuning these bright states are organized in a homoclinic snaking bifurcation structure. We describe how this bifurcation structure is destroyed when the detuning is increased across a critical value, and determine how a new bifurcation structure known as foliated snaking emerges.

Minisymposium 9	Monday, 17:00-17:30, Seminar room 2.066 (20.30)

Global bifurcation results for the Lugiato-Lefever equation

R. Mandel[1], **W. Reichel**[2]

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[2] *KIT, Institute for Analysis, Karlsruhe, Germany*

In this talk we discuss stationary and spatially 2π-periodic solutions of the Lugiato-Lefever equation

$$-da'' = (i - \zeta)a + |a|^2 a - if.$$

We prove that nontrivial frequency combs can only be observed for special ranges of values of the forcing and detuning parameters f and ζ, as it has been previously documented in experiments and numerical simulations. For instance, if the detuning parameter ζ is too large, then nontrivial frequency combs cease to exist. Additionally, we provide existence results for 2π-periodic solutions based on global bifurcation theory. It is illustrated that the bifurcating branches contain spatially concentrated solutions (bright or dark solitons) that also occur as final states of a dynamical detuning procedure.

This research is joint work with W. Reichel (KIT), see [1].

References

[1] R. Mandel, W. Reichel, A priori bounds and global bifurcation results for frequency combs modeled by the Lugiato-Lefever equation *SIAM J. Appl. Math.* **77** (2017), no. 1, 315–345.

Minisymposium 9	Monday, 17:30-18:00, Seminar room 2.066 (20.30)

Ultrafast optical ranging using microresonator soliton frequency combs

Philipp Trocha[1,*], **Maxim Karpov**[2], **Denis Ganin**[1], **Martin H. P. Pfeiffer**[2], **Arne Kordts**[2], **Jonas Krockenberger**[3], **Stefan Wolf**[1], **Pablo Marin-Palomo**[1], **Claudius Weimann**[4], **Sebastian Randel**[1], **Wolfgang Freude**[1,5], **Tobias J. Kippenberg**[2], **Christian Koos**[1,5]

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Microresonator-based dissipative Kerr solitons (DKS) show strong performance in applications such as frequency metrology [1], optical communications [2] and spectroscopy [3]. Here, optical ranging is presented yet as another field where two DKS frequency combs are used in a multi-heterodyne interferometric scheme. We achieve sampling rates of 100 MHz while maintaining a sub-μm precision, allowing surface sampling of objects moving at a speed of 150 m/s.

References

[1] J. D. Jost *et al.*, *Optica* **2** (2015), pp. 706–711.

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Minisymposium 9	Monday, 18:00-18:30, Seminar room 2.066 (20.30)

Dark and bright solitons in models for frequency combs

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Frequency combs are described as 2π-periodic solutions $a : \mathbb{R} \to \mathbb{C}$ of

$$-da'' + (\zeta - \mathrm{i})a - |a|^2 a + \mathrm{i}f = 0 \qquad \text{(LLE)}$$

for $d, \zeta, f \in \mathbb{R}$. Our analytical investigation of soliton combs is done in three steps:

- (LLE) without forcing and without damping: $-da'' + \zeta a - |a|^2 a = 0$
- (LLE) with forcing and without damping: $-da'' + \zeta a - |a|^2 a + \mathrm{i}f = 0$,
- (LLE) with forcing and with damping: $-da'' + (\zeta - \mathrm{i})a - |a|^2 a + \mathrm{i}f = 0$.

This approach can also be used to get soliton solutions via numerical path continuation in the software package pde2path.

In [1], frequency combs on continua which bifurcate from curves of trivial frequency combs are found. Here, we show how to numerically compute such frequency combs for a large range of parameters using pde2path and investigate quality measures such as combwidth and power conversion efficiency.

Additionally, as an extension of the model including two-photon-absorption, we consider (LLE) with higher order damping $\kappa > 0$

$$-da'' + (\zeta - \mathrm{i})a - (1 + \mathrm{i}\kappa)|a|^2 a + \mathrm{i}f = 0.$$

In this setting, we prove that there exists a value $\kappa^\star$, such that for $0 < \kappa < \kappa^\star$ soliton solutions persist but for $\kappa > \kappa^\star$ all non-trivial solutions disappear.

References

[1] R. Mandel and W. Reichel, A priori bounds and global bifurcation results for frequency combs modeled by the Lugiato Lefever equation, *SIAM J. Appl. Math.* **77** (2017).

Minisymposium 10 **Tuesday, 11:30-12:00, Seminar room 2.066 (20.30)**

Extrapolation of solutions of wave equations in the frequency domain: a microlocal viewpoint

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This talk considers the question of frequency extrapolation of bandlimited recordings of scattered waves. Much recent work on this question involves tools from signal processing and optimization, but there is a more "pleasing" alternative where microlocal analysis plays a central role. The method operates in a model-extended domain, where redundancy of the data is key. Frequency extrapolation is helpful, for the most part, because it can help bootstrap the frequency sweeps for full waveform inversion. Joint work with Yunyue Elita Li.

Minisymposium 10 **Tuesday, 12:00-12:30, Seminar room 2.066 (20.30)**

Microlocal analysis and numerical schemes for time harmonic waves

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The Helmholtz equation in an inhomogeneous medium is difficult to solve numerically. Various competing methods exist, but even while a number of researchers made significant progress in recent years, the cost to solve the problem in large 3-D domains is still relatively high. In this presentation I will present some recent results concerning this problem which have been strongly motivated by concepts from microlocal and WKB analysis. In particular, we will show that the consideration of discrete WKB type solutions can lead to high quality numerical schemes.

Minisymposium 10 **Tuesday, 12:30-13:00, Seminar room 2.066 (20.30)**

Bilinear operators and Fréchet differentiability in seismic imaging

Allan Greenleaf[1,*], **Margaret Cheney**[2], **Raluca Felea**[3], **Romina Gaburro and Cliff Nolan**[4]
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High frequency linearized seismic imaging uses microlocal analysis to understand how singularities in the sound speed can be recovered from seismic experiments. A number of authors have shown how multipathing combines with the type of seismic data (e.g., single source or marine) to result in artifacts in the reconstructions. Until recently the theoretical basis for this, namely the Fréchet differentiability of the map taking the sound speed to the data, was not known. Kirsch and Rieder [1] established this for certain pairs of function spaces; similar results are also special cases of [2]. This talk will describe preliminary work on trying to establish similar results for function spaces more adapted to the linearizations, which are degenerate Fourier integral operators, in the presence of multipathing.

References

[1] A. Kirsch and A. Rieder, On the linearization of operators related to the full waveform inversion in seismology, *Math. Methods Appl. Sci.* **37** (2014), pp. 2995–3007; Seismic tomography is locally ill-posed, *Inverse Problems* **30** (2014), 125001.

[2] K. Blazek, C. Stolk and W. Symes, A mathematical framework for inverse wave problems in heterogeneous media, *Inverse Problems* **29** (2013), 065001.

Minisymposium 10 **Tuesday, 15:00-15:30, Seminar room 2.066 (20.30)**

Micolocal analysis of Doppler SAR

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We consider the forward operator F which maps the image to the data, and arises in the Doppler Synthetic Aperture Radar (SAR) imaging. We study the microlocal properties of F in two different cases: the straight trajectory and the circular trajectory and we show that it exhibits blowdown, fold and cusp singularities. Next, we compare the results with the monostatic SAR imaging.

Minisymposium 10 **Tuesday, 15:30-16:00, Seminar room 2.066 (20.30)**

Microlocal analysis of a spindle transform arising in Compton scattering tomography

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A mathematical model for Compton scattering tomography involves a forward operator that gives integration over spindle tori. We provide a microlocal analysis of the forward operator and corresponding normal operator for Compton scattering tomography in a specific scanning geometry proposed in [1]. The analysis shows that the Schwarz kernel of the normal operator is in a class of distributions associated to paired Lagrangians. This characterisation allows us to predict the presence of spherical artefacts in reconstructions, and we confirm the presence of these artefacts in numerical experiments from simulated data. This talk is based on [2].

References

[1] J. Webber and W. Lionheart, Three dimensional Compton scattering tomography, *preprint arXiv:1704.03378*, 2017.

[2] J. Webber and S. Holman, Microlocal analysis of a spindle transform, *preprint arXiv:1706.03168*, 2017.

Minisymposium 10 **Tuesday, 16:30-17:00, Seminar room 2.066 (20.30)**

Local and global boundary rigidity

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The boundary rigidity problem consist of recovering a Riemannian metric in a domain, up to an isometry, from the distance between boundary points. We show that in dimensions three and higher, knowing the distance near a fixed strictly convex boundary point allows us to reconstruct the metric inside the domain near that point, and that this reconstruction is stable. We also prove semi-global and global results under certain an assumption of the existence of a strictly convex foliation. The problem can be reformulated as a recovery of the metric from arrival times of waves between boundary points; which is known as travel-time tomography. The interest in this problem is motivated by seismology: to recover the sub-surface structure of the Earth given travel-times from the propagation of seismic waves. In oil exploration, the "seismic signals" are artificially created and the problem is local in nature. In particular, we can recover locally the compressional and the shear wave speeds for the elastic Earth model, given local information. The talk is based on joint work with G. Uhlmann and A. Vasy.

References

[1] P. Stefanov, G. Uhlmann and A. Vasy, Boundary rigidity with partial data, *J. Amer. Math. Soc.*, 2016, 29, 299–332.

[2] P. Stefanov, G. Uhlmann and A. Vasy, Local and global boundary rigidity and the geodesic X-ray transform in the normal gauge, 2017, arXiv:1702.03638.

[3] P. Stefanov, G. Uhlmann and A. Vasy, Local recovery of the compressional and shear speeds from the hyperbolic DN map, *Inverse Problems*, 2018, 34 014003.

Minisymposium 10 **Tuesday, 17:00-17:30, Seminar room 2.066 (20.30)**

Microlocal methods for geodesic X-ray transforms

Francois Monard[1,*]

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In this talk, we will discuss the inverse problem of reconstructing a function from its integrals along a collection of geodesic curves for a fixed Riemannian metric (namely, its geodesic X-ray transform). As is now well-documented, the reconstruction problem is sensitive to certain geometric features of the associated metric (e.g., conjugate points, trapped set), and microlocal methods enter the picture to produce positive or negative answers depending on the context. Time allowing, we will discuss the following topics:

(i) Sharp mapping properties for the normal operator in simple geometries and applications to Uncertainty Quantification for X-ray transforms. Based on [1].

(ii) Artifact-generating FIO's and loss of stability in geometries with conjugate points. Based on [2,3].

References

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Minisymposium 10 **Tuesday, 17:30-18:00, Seminar room 2.066 (20.30)**

Microlocal inversion of certain restricted ray transforms of symmetric tensor fields

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We consider the integral geometry problem of recovering rank m symmetric tensor fields from its integrals along lines in n-dimensional space. We focus on longitudinal and transverse ray transforms restricted to lines passing through a fixed smooth curve. Under suitable conditions on the curve, we present microlocal inversion results.

Minisymposium 10 **Wednesday, 15:00-15:30, Seminar room 2.067 (20.30)**

An explicit method of reconstruction for X-ray phase contrast imaging

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The method of x-ray phase contrast imaging in the near-field propagation regime is effective for microscale imaging of biological tissue. Using coherent hard X-rays this method permits simultaneous phase retrieval and tomographic inversion with improved contrast but involves a phase retrieval problem since of physical limitation of detectors. An explicit formulas are proposed for reconstruction of the complex refraction index of an object from intensity distribution of one hologram [1]. The method is based on the classical complex analysis.

References

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Minisymposium 10 **Wednesday, 15:30-16:00, Seminar room 2.067 (20.30)**

Wavelet-based reconstructions in limited data photoacoustic tomography

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In this talk, we will investigate a reconstruction problem that arises in limited data photoacoustic tomography with a flat observation surface. In the first part of this talk, we will explain which singularities of the original object can be reliably reconstructed and why artifacts can be generated when applying the classical FBP-type reconstruction operators to limited data. We will also provide precise characterizations of added artifacts and explain how they can be reduced. In the second part of this talk, we will present a stable reconstruction method, which is based on sparsity assumptions in the wavelet domain. In particular, we will present an easy to implement numerical algorithm for that problem. This talk presents results from a joint work with Eric Todd Quinto (Tufts University) and Markus Haltmeier (University of Innsbruck).

Minisymposium 10 **Wednesday, 16:30-17:00, Seminar room 2.067 (20.30)**

Analysis of reconstruction methods for photoacoustic tomography in heterogenous media

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We analyse the inverse problem of photoacoustic tomography (PAT) with spatially variable sound speed and damping in an L^2-setting as well as an H^1-setting. The adjoint operators are casted in the form of a nonstandard wave equation for which we derive well-posedness and finite speed of propagation results. We analyze the normal operator from a microlocal analysis point of view. For the inverse problem of PAT, under the visibility condition, standard iterative schemes are proven to achieve a linear rate of convergence. If the visibility condition is not satisfied, regularization methods including total variation (TV) regularization are employed. In the full data case, our simulations demonstrate that the CG method is fast and robust. In the ill-posed case, TV regularization is shown to significantly improve the reconstruction quality. The talk is based on the recent works [1,2].

References

[1] M. Haltmeier and L. V. Nguyen, Analysis of iterative methods in photoacoustic tomography with variable sound speed, *SIAM J. Imaging Sci.* **10** (2017), pp. 751–781.

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Minisymposium 10 **Wednesday, 17:00-17:30, Seminar room 2.067 (20.30)**

Microlocal analysis of dynamic imaging problems

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The reconstruction process in tomography is well understood if the specimen is stationary during data acquisition. However, structural and functional changes of a specimen, e.g. in medical imaging or non-destructive testing, lead to inconsistent data and, if the dynamic behaviour is not appropriately taken into account, to motion artefacts in the reconstructed images.

The effect of the dynamics can be studied using methods from microlocal analysis. By analysing microlocal properties of forward and reconstruction operator, we can characterize the visible singularities of the object as well as additional artefacts arising in the reconstruction [1,2]. In this talk, we will discuss several scenarios, ranging from exact to inaccurate motion information and from global to local deformations. These studies provide valuable insights facilitating a further image analysis and/or culminate in computational methods for artefact reduction.

References

[1] B. N. Hahn and E. T. Quinto, Detectable singularities from dynamic Radon data, *SIAM J. Imaging Sci.* **9** (2016), pp. 1195–1225.

[2] B. N. Hahn, A motion artefact study and locally deforming objects in computerized tomography *Inverse Problems* **33** (2017), 114001.

Minisymposium 11 — **Wednesday, 15:00-15:30, Seminar room 0.014 (20.30)**

Computation and stability of waves in Hamiltonian PDEs

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Nonlinear waves in Hamiltonian PDEs are known to be quite sensitive to perturbations. Typically, such waves come in families (relative equilibria) due to the invariance of the Hamiltonian with respect to some group action, and perturbations lead to interference patterns which persist over long time intervals. In their seminal work, Grillakis, Shatah and Strauss provide sufficient conditions on the Hamiltonian which guarantee orbital Lyapunov stability of a wave with respect to suitable Sobolev norms. In this contribution we show that the freezing method is a suitable tool for solving the Cauchy problem near such nonlinear waves. By adding phase conditions, the method is based on a transformation of the original PDE into a partial differential algebraic equation (PDAE), which is then solved numerically. In this way, relative equilibria of the PDE become steady states of the PDAE, and nearby solutions are split into the motion of a profile and of a variable in the underlying group. We prove that classical Lyapunov stability holds for the PDAE under the assumptions of the Grillakis-Shatah-Strauss theory. We also analyze numerical errors caused by spatial discretization and show applications to the nonlinear Schrödinger and the nonlinear Klein-Gordon equation.

The results are based on the PhD thesis [1] of the second author.

References

[1] S. Dieckmann, *Dynamics of patterns in equivariant Hamiltonian partial differential equations.* PhD thesis, Bielefeld University, 2017.

Minisymposium 11 — **Wednesday, 15:30-16:00, Seminar room 0.014 (20.30)**

Traveling waves in highly nonlinear shallow water equations

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Motivated by the question whether higher-order nonlinear model equations, which go beyond the Camassa-Holm regime of moderate amplitude waves, could point us to new types of waves profiles, we study the traveling wave solutions of a quasilinear evolution equation which models the propagation of shallow water waves of *large amplitude*. Apart from symmetric smooth, peaked and cusped solitary and periodic traveling waves, whose existence is well-known for moderate amplitude equations like Camassa-Holm, we obtain entirely new types of *singular traveling waves*: periodic waves which exhibit singularities on both the crest and the trough simultaneously, waves with asymmetric peaks, as well as multi-crested smooth and multi-peaked waves with decay. Our approach uses qualitative tools for dynamical systems and methods for integrable planar systems.

References

[1] A. Geyer, R. Quirchmayr, Traveling wave solutions of a highly nonlinear shallow water equation, Discret. Contin. Dyn. Syst. Ser. A. 38 (2018).

Minisymposium 11 Thursday, 11:30-12:00, Seminar room 1.067 (20.30)

Rotating waves driven by heating in spherical shells

Juan Sánchez Umbría[1,*], **Ferran Garcia Gonzàlez**[2], **Marta Net**[1]
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A methodology to compute rotating (RW) and modulated rotating waves (MRW), arising in thermal convection of fluids confined in spherical shells, and their stability, will be presented. It is based on continuation, Newton Krylov and Arnoldi methods. The MRW are calculated as periodic orbits of the Navier-Stokes equations, written in the rotating frame of reference where the only time dependence is the modulation of the waves. The parameters of the study are selected in an interval of multistability of the rotating waves due to the proximity of a double-Hopf bifurcation. The MRW are stable in a short range of Rayleigh numbers, and lose stability at a third Hopf bifurcation giving rise to three-frequency stable solutions. However, the computation of unstable RW and MRW allowed to see that the nontrivial dynamics found by time integration is close to heteroclinic chains visiting both types of states.

References

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Minisymposium 11 Thursday, 12:00-12:30, Seminar room 1.067 (20.30)

The Role of Self-Organized Spatial Patterns in the Design of Agroforestry Systems

Omer Tzuk[1,*], **Hannes Uecker**[2], **Pedro Berliner**[3], **Ehud Meron**[1]
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[2]*Department of Mathematics, Universität Oldenburg, Oldenburg, Germany*
[3]*Jacob Blaustein Institute for Desert Research, Sede-Boqer Campus, Israel*
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The development of sustainable agricultural systems in drylands is currently an important issue in the context of mitigating the outcomes of population growth under the conditions of climatic changes. The need to meet the growing demand for food, fodder, and fuel under the threat of climate change, requires cross-disciplinary studies of ways to increase the livelihood while minimizing the impact on the environment. Practices of agroforestry systems, in which herbaceous species are intercropped between rows of woody species plantations, have shown to increase land productivity. As vegetation in drylands tends to self-organize in spatial patterns, it is important to explore the relationship between the patterns that agroforestry systems tend to form, and the productivity of these system in terms of biomass, their resilience to droughts, and water use efficiency.

A spatially-explicit vegetation model for two species that compete for water and light and may exploit soil layers of different depths will be introduced. Spatially-uniform and periodic solutions, and their stability properties, will be presented for different scenarios of species and environmental conditions. The implications for optimal intercropping in terms of biomass productivity, water use efficiency, and resilience to environmental changes, will be discussed.

Minisymposium 11 **Thursday, 12:30-13:00, Seminar room 1.067 (20.30)**

Patterns and Waves in Nonlocal Reaction-Diffusion Equations

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In this talk, I shall present several recent results on reaction-diffusion equations involving nonlocal terms. We focus on quadratic and cubic nonlinearities present in the classical Fisher-KPP and Nagumo/Allen-Cahn/Ginzburg-Landau equations respectively. The nonlocal contributions arise due to (potentially singular) spatial convolution or integral terms replacing the usual local differential operators and nonlinearities. The dynamical systems viewpoint I shall present considers: stationary solutions, amplitude normal forms for bifurcations, and travelling wave patterns for several benchmark example problems. Detailed results can be found in the series of papers [1,2,3,4].

References

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Minisymposium 11 **Thursday, 15:00-15:30, Seminar room 1.067 (20.30)**

A network of invariant solutions in inclined layer convection

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Thermal convection in a fluid between two horizontal plates, a lower hot plate and an upper cold plate, exhibits chaotic dynamics and turbulence. If such a convection cell is inclined against gravity, buoyancy force drives hot and cold fluid up and down the incline leading to a shear flow in the base state and the emergence of complex dynamics and spatio-temporal patterns. We study the dynamics of inclined layer convection (ILC) numerically and present a collection of invariant solutions of ILC, including fixed points, travelling waves, periodic orbits and heteroclinic orbits. At intermediate angles of inclination, a simple network of invariant solutions may guides moderately complex dynamics. The complexity the dynamics increases with the intensity of the shear flow in ILC. By controlling the shear flow intensity via angle of inclination or thermal driving, ILC offers a systematic approach to study shear flow dynamics in phase space.

Minisymposium 11 **Thursday, 15:30-16:00, Seminar room 1.067 (20.30)**

Front propagation in bistable pattern-forming systems

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We discuss and analyze front propagation in weakly subcritical pattern-forming systems described by the amplitude equation [1]

$$A_t = A_{xx} + \mu A + ia_1|A|^2 A_x + ia_2 A^2 \bar{A}_x + |A|^2 A - |A|^4 A,$$

where μ is the bifurcation parameter and the coefficients a_1 and a_2 are real. Exact front solutions are found and their linear stability analyzed as a function of the parameters. In some cases the exact solutions are stable but are not selected from arbitrary small amplitude initial conditions. Instabilities select distinct fronts, including double fronts and chaotic fronts. We also study stationary localized structures or gap solitons created via spatial pinning described by [2]

$$-iA_t = A_{xx} - V(x)A + |A|^2 A - |A|^4 A,$$

where $V(x)$ is a given periodic potential, and investigate their dissolution under perturbations with finite energy and phase gradient. In some cases the fronts bounding the structure unpin and the structure may expand or travel, depositing energy in potential wells as it does so.

References

[1] B.C. Ponedel, H.-C. Kao and E. Knobloch, Front propagation in weakly subcritical pattern-forming systems, *Phys. Rev. E* **96** (2017) 032208.

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Minisymposium 11 **Thursday, 16:30-17:00, Seminar room 1.067 (20.30)**

Wave-pinning, Turing instability and localised pattern formation; three sides of the same coin?

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Recent work will be described that on systems of reaction diffusion equations of Schnakenberg-type. The original motivation is in the study of the dynamics of small G-protiens known as ROPs that are responsible for polarity formation across all forms of eukaryotic cells. The active form diffuses more slowly as it is constrained to the cell membrane whereas the inactive form is free to diffuse in the cytosol. A subcritical Turing bifurcation gives the onset of localised patterns, which transform into spike-like solutions in another part of the domain. In this talk I shall give three recent updates to the paper we published last year. First, an imporvement to the asymptotic study shows how in the singular limit of protein conservation, the spikes become fronts in a non-trivial way, recovering the so-called wave-pinning dynamics. Second, the mechanism for localised pattern to spike transition is studied using a Shilnikov-type analysis. The transition is argued to be universal and examples are found in a models of vegetation patterning, nonlinear optics and crime-wave hotspots. Finally, preliminary results are presented on how the protein dynamics feeds back into cell shape morphogenesis.

References

[1] N. Verschueren and A. Champneys A Model for Cell Polarization Without Mass Conservation *SIAM J. APPl. Dynamical Sys.* **16** (2017), pp. 1797–1830.

Minisymposium 11 **Thursday, 17:00-17:30, Seminar room 1.067 (20.30)**

Spatial solitons in PT-symmetric systems: bifurcation from eigenvalues and from spectral intervals

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In PT-symmetric nonlinear systems bound states with real propagation constants (i.e. spatial solitons) have been previously found numerically. We present a proof of their bifurcation in two important cases: bifurcation from a simple eigenvalue and from a spectral interval. In the case of a simple eigenvalue we work in the general abstract setting $A\psi - \varepsilon f(\psi) = \lambda\psi$, with $\varepsilon \in \mathbb{R}$ (small), a Lipschitz continuous f and with a closed linear operator A densely defined on a Hilbert space and having a non-empty resolvent set and possessing an antilinear symmetry.

In the more difficult case of the bifurcation from a spectral interval we consider the one dimensional stationary Gross-Pitaevskii (GP) equation $-\psi''+V(x)\psi+\sigma(x)|\psi|^2\psi = \lambda\psi,\ x \in \mathbb{R}$, with periodic PT-symmetric (generally complex) coefficients V and σ.

We provide explicit asymptotic expansions of the bound states with error estimates.

References

[1] T. D. and P. Siegl., Bifurcation of nonlinear eigenvalues in problems with an antilinear symmetry, *J. Math. Phys* **57** (2016), 093502.

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Minisymposium 11 **Thursday, 17:30-18:00, Seminar room 1.067 (20.30)**

Modulation equations at the Eckhaus boundary – The KdV equation –

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We are interested in the description of small modulations in time and space of wave-train solutions of the complex Ginzburg-Landau equation

$$\partial_T A = (1 + i\alpha)\partial_X^2 A + A - (1 + i\beta)A|A|^2$$

near the Eckhaus boundary. Depending on the real parameters α, β a number of modulation equations, such as the KdV equation or a generalized Ginzburg-Landau equation systems can be derived [1]. In this talk some error estimates will be proven showing that the KdV approximation makes correct predictions in a certain parameter regime where beyond the Eckhaus boundary a small side-band instability appears.

References

[1] A. van Harten, Modulated modulation equations, *Proceedings of the IUTAM/ISIMM Symposium on Structure and Dynamics of Nonlinear Waves in Fluids, Hannover, Germany, 1994*, pp. 117130.

Minisymposium 11 **Thursday, 18:00-18:30, Seminar room 1.067 (20.30)**

Localized structures in an extended Klausmeier model

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The evolution of vegetation patterns in semiarid regions can be described by a system of reaction-diffusion equations for biomass and water. Despite its simplicity this model features many of the pattern forming phenomena observed in nature and presents a valuable tool to understand the formation and reversing of desertification. We show an extension of the model where the variation of the terrain is taken into account via spatially varying coefficients. Using a novel mix of geometric singular perturbation theory and exponential dichotomies, we show a new construction of localized vegetation patches and discuss their stability and interactions. This is joint work with Arjen Doelman and Robbin Bastiaansen.

Minisymposium 12	Tuesday, 11:30-12:00, Seminar room 1.067 (20.30)

One-dimensional periodic solutions in a three-component reaction-diffusion system

Gianne Derks[1,*], **Peter van Heijster**[2], **David J.B. Lloyd**[1]

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Periodic patterns occur ubiquitously in nature, but the mechanism behind the formation of periodic patterns away from onset is not well understood. In this talk we consider the mechanism behind the generation of periodic stationary solutions in a singularly perturbed reaction-diffusion system. The system has one fast nonlinear component, interacting with two slow components. We investigate the existence and bifurcations of families of one-dimensional periodic solutions in this system. It will be shown how changes in the slow manifold and changes in the fast dynamics lead to an intriguing sequence of self-replicating patterns of large amplitude periodic waves. We will conclude this talk with a discussion about extensions to two-dimensional patterns and travelling waves.

Minisymposium 12	Tuesday, 12:00-12:30, Seminar room 1.067 (20.30)

Striped pattern selection by advective reaction-diffusion systems

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We study a Gray-Scott type system of PDEs that models vegetation in drylands. Remote-sense observations of striped vegetation patterns along slope contours have been widespread. For a general class of systems we perform a linear stability analysis showing that destabilization of a spatially homogeneous state occurs through perturbations in the direction of advection, leading to striped pattern formation. Concerning striped pattern stability, we numerically study the breakup into rectangles or rhombs. It is shown that an increase in slope/advection leads to an increased resilience of vegetation stripes. On flat terrain, vegetation stripes with a large wavelength are never (transversally) stable.

References

[1] E. Siero and A. Doelman and M.B. Eppinga and J.D.M. Rademacher and M. Rietkerk and K. Siteur, Striped pattern selection by advective reaction-diffusion systems: Resilience of banded vegetation on slopes, *Chaos* **25** (2015)

Minisymposium 12 **Tuesday, 12:30-13:00, Seminar room 1.067 (20.30)**

Grain boundaries for the Bénard-Rayleigh problem

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We study the existence of grain boundaries for the stationary Bénard-Rayleigh system. The analysis relies upon a spatial dynamics formulation of the existence problem and a centre-manifold reduction. In this setting, the grain boundaries are found as heteroclinic orbits of a reduced system of ODEs. A normal form transformation allows us to identify a leading-order approximation, which is then shown to persist using transversality arguments.

Minisymposium 12 **Tuesday, 16:30-17:00, Seminar room 1.067 (20.30)**

Pattern formation in the wake of growth mechanisms

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Externally mediated, or "triggered," spatial patterns have become a topic of recent interest in many fields, such as directional quenching in alloy melts, growing interfaces in biological systems, moving masks in ion milling, and traveling reaction fronts. Mathematically, they can be encoded in a step-like parameter dependence that allows patterns in a half plane, and suppresses them in the complement, while the boundary of the pattern-forming region propagates with fixed normal velocity.

In this talk, I will show how techniques from dynamical systems, functional analysis, and numerical continuation, can be used to study the effect of these traveling in-homogeneities on patterns left in the wake; finding for example how the speed of the parameter interface affects orientation and deformation of striped patterns. I will explain this approach in the context of the Swift-Hohenberg PDE, a prototypical model for many pattern forming systems, posed in one and two spatial dimensions. I will also discuss recent work studying the stability of these structures, their dynamics via a modulational approach, and the extension and application of our approach to other prototypical models, such as the complex Ginzburg-Landau equation, and reaction-diffusion systems.

Minisymposium 12	Tuesday, 17:00-17:30, Seminar room 1.067 (20.30)

Beyond all order asymptotics for homoclinic snaking in a Schnakenberg system

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We investigate localized patterns in an one dimensional Schnakenberg system via beyond all order methods. We derive a (divergent) asymptotic series of a front solution, which is in a 1:1 correspondense to localized patterns, and show, by optimal truncation, that the remainder is uniformly exponentially small if and only if a specific parameter range is met.

Minisymposium 12	Tuesday, 17:30-18:00, Seminar room 1.067 (20.30)

Modulated traveling fronts for the Swift-Hohenberg equation in case of an additional conservation law

Bastian Hilder[1,*]
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We consider the cubic Swift-Hohenberg equation coupled with a conservation law,

$$\partial_t u = -(1+\partial_x^2)^2 u + \alpha u + uv - u^3,$$
$$\partial_t v = \partial_x^2 v + \gamma \partial_x^2 (u^2).$$

This system has a family of spatially homogeneous equilibria which each get unstable as α increases beyond a critical value. Near this first instability stationary periodic solutions emerge [3]. We show the existence of front solutions connecting the ground state with these patterns and thus present a mechanism of pattern formation. The proof uses center manifold theory to reduce the problem to an effective, finite dimensional ODE system following [1,2].

References

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Minisymposium 13 **Monday, 11:45-12:15, Seminar room -1.025 (20.30)**

On stability of blow up solutions for the critical co-rotational Wave Maps problem

Joachim Krieger[1], **Shuang Miao**[2,*]
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In this talk, I will present our recent work on the stability of type II blow up solutions for the critical co-rotational wave maps equation. We show that the solution constructed in [1,2] are stable under small perturbations within co-rotational class, provided that the scaling parameter $\lambda(t) = t^{-1-\nu}$ is sufficiently close to t^{-1}, i.e. the constant $\nu > 0$ is sufficiently small. This is a joint work with Joachim Krieger.

References

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Minisymposium 13 **Monday, 12:15-12:45, Seminar room -1.025 (20.30)**

Existence and stability of blowup for wave maps into negatively curved targets

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We consider wave maps from $(1+d)$-dimensional Minkowski space into negatively curved target manifolds. Furthermore, we concern ourselves with the question of blowup. In [1] it was proved that self-similar blowup exists for $d = 7$. We complement this result by constructing for each dimension $d \geq 8$ a negatively curved, $d-$dimensional target manifold that allows for the existence of a self-similar wave map. What is more, our solutions provide a stable blowup mechanism for the corresponding Cauchy problem. This is the first example of stable blowup for wave maps with negatively curved targets.

References

[1] T. Cazenave, J. Shatah, and S. Tahvildar-Zadeh. Harmonic maps of the hyperbolic space and development of singularities in wave maps and Yang-Mills fields. *Ann. Inst. H. Poincaré Phys. Théor.*, 68(3): 315–349, 1998.

Minisymposium 13	Monday, 12:45-13:15, Seminar room -1.025 (20.30)

Dynamics of strongly interacting unstable two-solitons for generalized Korteweg-de Vries equations

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Many evolution PDEs admit special solutions, called solitons, whose shape does not change in time. A multi-soliton is a solution which is close to a superposition of a finite number K of solitons placed at a large distance from each other. I am interested in describing multi-soliton dynamics for generalized Korteweg-de Vries equations. I will present a general method of formally predicting the time evolution of the centers and velocities of each soliton. Then I will discuss in detail the case $K = 2$, in particular in the regime of strong interactions, which occurs when the velocities of both solitons converge to the same value for large times. Under the additional assumption that the solitons are linearly unstable, one can show that the formal method correctly predicts the distance between the solitons for large times.

Minisymposium 13	Monday, 15:00:-15:30, Seminar room -1.025 (20.30)

Singularity formation for Burgers equation with transversal viscosity and related problems

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This talk is about singularity formation for solutions to

$$(*)\quad \partial_t u + u\partial_x u - \partial_{yy} u = 0,\ (x, y) \in \mathbb{R}^2$$

which is a simplified model of Prandtl's boundary layer equation. We construct precise singular solutions which form a shock along the streamwise variable x, whose scaling parameters evolve according to a coupled parabolic system along the transversal y variable. We will first investigate the self-similarity in the shocks of Burgers equation, and then understand the 2-dimensional problem via precise asymptotic for solutions to the parabolic system. We will then see how this can be related to the blow-up problem for Prandtl's equations, and mention other anisotropic blow-up issues.

References

[1] C. Collot, T.-E. Ghoul, N. Masmoudi, Singularity formation for Burgers equation with transversal viscosity, *submitted* (2018).

Minisymposium 13 **Monday, 15:30-16:00, Seminar room -1.025 (20.30)**

Stable Self-Similar Blowup for a family of nonlocal transport equations

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The difference between the two-dimensional and three-dimensional Euler equation is the presence of the so-called vortex-stretching term $\omega \cdot \nabla u$. In 1985 Constantin, Lax and Majda introduced a 1D model to understand the effect of the vortex streching, $\partial_t \omega = H(\omega)\omega$, with $H(\omega)(x) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{\omega(y)}{x-y} dy$. They solve explicitly the equation and prove that it blows up in finite time, De Gregorio introduced a model that take into account both effect vortex stretching and transport, by adding an extra term $-\Lambda^{-1}\omega\partial_x\omega$ with $\Lambda^{-1}\omega(x) = -\int_0^x H\omega(t)dt$. De Gregorio conjectured by using numerical simulations that the addition of the transport term should lead to global regularity. Inspired from this conjecture Okamoto, Sakajo, and Wunsch introduced a new model where they weight the transport term with a parameter a.

$$\partial_t \omega - a\Lambda^{-1}\omega\partial_x\omega = 2H(\omega)\omega(x,t) \in \mathbb{R} \times [0, T_*).$$

Hence, when $a = 2$ we get the De Gregorio model and when $a = 0$ we get CLM model. In the same idea of Constantin, Lax, and Majda Cordoba, Cordoba and Fontelos introduced a 1D model to mimic the 2D quasi-geostrophic equation:

which correspond to $a = -2$ for the OSW model. Recently Elgindi and Jeong proved the existence of an analytic self-similar profile for a small by using a local bifucartion argument. The goal here is to prove the stability of those profile for all a small enough.

Minisymposium 13 **Monday, 16:30-17:00, Seminar room -1.025 (20.30)**

Global attraction to solitary waves for Klein-Gordon equation with concentrated nonlinearities

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The global attraction is proved for the nonlinear 3D Klein-Gordon equation coupled to several nonlinear point oscillators. Our main result is the convergence of each finite energy solution to the set of all solitary waves as $t \to \pm\infty$. This attraction is caused by the nonlinear energy transfer from lower harmonics to the continuous spectrum and subsequent dispersion radiation.

We justify this mechanism by the following strategy based on *inflation of spectrum by the nonlinearity*. We show that any *omega-limit trajectory* has the time-spectrum in the spectral gap $[-m, m]$ and satisfies the original equation. Then the application of the Titchmarsh convolution theorem reduces the spectrum of each omega-limit trajectory to a single frequency.

References

[1] E. Kopylova, On global well-posedness for Klein–Gordon equation with concentrated nonlinearities, *J. Math. Anal. Appl.* **443** (2016), no. 2, pp. 1142–1157.

[2] E. Kopylova, On global attraction to solitary waves for the Klein–Gordon equation with concentrated nonlinearity, *Nonlinearity* **30** (2017), pp. 4191–4207

[3] E. Kopylova and A. Komech, On global attractor to solitary waves for 3D Klein–Gordon equation with several concentrated nonlinearities, submitted to *Dynamics of Partial Differential Equations*

Minisymposium 13 **Monday, 17:00-17:30, Seminar room -1.025 (20.30)**

Type II blow up solutions with optimal stability properties for the critical focussing nonlinear wave equation on $\mathbb{R}^{3+1}$.

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We show that the finite time type II blow up solutions for the energy critical nonlinear wave equation

$$\Box u = -u^5$$

on $\mathbb{R}^{3+1}$ constructed in [1], [2] are stable along a co-dimension one Lipschitz manifold of data perturbations in a suitable topology, provided the scaling parameter $\lambda(t) = t^{-1-\nu}$ is sufficiently close to the self-similar rate, i. e. $\nu > 0$ is sufficiently small. This result is qualitatively optimal in light of the result of [3]. The paper builds on the analysis of [4].

References

[1] J. Krieger, W. Schlag, D. Tataru, Slow blow-up solutions for the $H^1(\mathbb{R}^3)$ critical focusing semilinear wave equation, *Duke Math. J.,* **147** (2009), pp. 1–53.

[2] J. Krieger, W. Schlag, Full range of blow up exponents for the quintic wave equation in three dimensions, *J. Math. Pures Appl. (9),* **101** (2014), pp. 873–900.

[3] J. Krieger, K. Nakanishi, W. Schlag, Center-stable manifold of the ground state in the energy space for the critical wave equation, *Mathematische Annalen,* **361** (2015), pp. 1–50.

[4] J. Krieger, On stability of type II blow up for the critical NLW on $\mathbb{R}^{3+1}$, preprint 2017.

Minisymposium 13 **Monday, 17:30-18:00, Seminar room -1.025 (20.30)**

Breather solutions for nonlinear Klein-Gordon equations on periodic metric graphs

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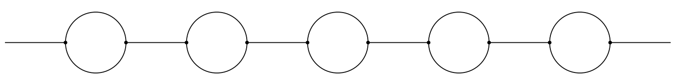
We consider a nonlinear Klein-Gordon equation posed on the periodic, metric necklace graph.



We prove the existence of time-periodic, real-valued and spatially localized solutions of small amplitude. In general, the existence of such breather solutions is very rare, cf. [2]. Our result builds upon a spatial dynamics ansatz combined with center manifold reduction and bifurcation theory. We essentially use gaps in the spectral picture of the linear operator investigated by Floquet-Bloch theory. Our work is motivated by [1] and part of my PhD thesis supervised by Guido Schneider.

References

[1] C. Blank, M. Chirilus-Bruckner, V. Lescarret, and G. Schneider, Breather solutions in periodic media, *Comm. Math. Phys.*, 302(3):815-841, 2011.

[2] J. Denzler, Nonpersistence of breather families for the perturbed sine Gordon equation, *Comm. Math. Phys.*, 158(2):397-430, 1993.

Minisymposium 13 — **Monday, 18:00-18:30, Seminar room -1.025 (20.30)**

A priori estimates and existence of periodic solutions to the modified Benjamin-Ono equation below $H^{1/2}(\mathbb{T})$

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We discuss a priori estimates and the existence of solutions to the modified Benjamin-Ono equation (mBO)

$$\begin{cases} \partial_t u + \mathcal{H}\partial_{xx} u & = \partial_x(u^3/3), \ (x,t) \in \mathbb{K} \times \mathbb{R}, \\ u(0,x) & = u_0(x) \in H^s_{\mathbb{R}}(\mathbb{K}), \end{cases} \tag{1}$$

with $\mathbb{K} \in \{\mathbb{R}, \mathbb{T}\}$. Localization of time to small frequency-dependent time intervals recovers control of solutions at low regularities and yields a priori estimates and existence of solutions for $1/4 < s < 1/2$. Previously, this was carried out on the real line in [1]. We prove the same results for periodic solutions after observing that the localization to short time intervals recovers dispersive properties from Euclidean space. The strategy can also be adjusted to deal with periodic solutions to the modified Korteweg-de Vries equation (cf. [2]).

References

[1] Z. Guo, Local well-posedness and a priori bounds for the modified Benjamin-Ono equation, *Adv. Differential Equations* **16** (2011), no. 11-12, pp. 1087–1137.

[2] R. Schippa, On the existence of periodic solutions to the modified Korteweg-de Vries equation below $H^{1/2}(\mathbb{T})$, *arXiv e-prints*, arXiv:1711.09720.

Minisymposium 14 Tuesday, 11:30-12:00, Seminar room -1.025 (20.30)

On linear stability of bi-frequency solitary waves of the nonlinear Dirac equation

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We construct bi-frequency solitary waves of the nonlinear Dirac equation with the scalar self-interaction, known as the Soler model (with an arbitrary nonlinearity and in arbitrary dimension) and the Dirac–Klein–Gordon with Yukawa self-interaction. We consider the relation between $\pm 2\omega\mathrm{i}$ eigenvalues of the linearization at a solitary wave, Bogoliubov $\mathbf{SU}(1,1)$ symmetry, and the existence of bi-frequency solitary waves. We show that the spectral stability of these waves reduces to spectral stability of usual (one-frequency) solitary waves which was obtained in [1,2,3]. The results are presented in [4].

References

[1] Nabile Boussaïd and Andrew Comech, On spectral stability of the nonlinear Dirac equation, *Journal of Functional Analysis* **271** (2016), 1462–1524.

[2] Nabile Boussaäd and Andrew Comech, Nonrelativistic asymptotics of solitary waves in the Dirac equation with the Soler-type nonlinearity, *SIAM J. Math. Anal.* **49** no. 4 (2017), pp. 2527–2572.

[3] Nabile Boussaïd and Andrew Comech, Spectral stability of small amplitude solitary waves of the Dirac equation with the Soler-type nonlinearity, *arXiv* **1705.05481** (2017), pp. 1–44.

[4] Nabile Boussaïd and Andrew Comech, On linear stability of bi-frequency solitary waves in Soler and Dirac–Klein–Gordon models, *arXiv* **1711.05654** (2017), pp. 1–5.

Minisymposium 14 Tuesday, 12:00-12:30, Seminar room -1.025 (20.30)

On stability of solitary waves of the nonlinear Dirac equation in the non-relativistic limit

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We study the point spectrum of the linearization at a solitary wave solution to the nonlinear Dirac equation in $\mathbb{R}^n$, for all $n \geq 1$, with Soler type power nonlinear term. We focus on the spectral stability, that is, the absence of eigenvalues with positive real part, in the nonrelativistic limit.

We prove the spectral stability of small amplitude solitary waves for the charge-subcritical cases and for the “charge-critical case”.

An important part of the stability analysis is the proof of the absence of bifurcations of nonzero-real-part eigenvalues from the embedded threshold points. Our approach is based on constructing a new family of exact bi-frequency solitary wave solutions in the Soler model and on the analysis of the behaviour of “nonlinear eigenvalues”.

The results are presented in [2]. They complement those from [1].

References

[1] Nabile Boussaïd and Andrew Comech, On spectral stability of the nonlinear Dirac equation, *Journal of Functional Analysis* **271** (2016), 1462–1524.

[2] Nabile Boussaïd and Andrew Comech, Spectral stability of small amplitude solitary waves of the Dirac equation with the Soler-type nonlinearity, *arXiv* **1705.05481** (2017), pp. 1–44.

Minisymposium 14 Tuesday, 12:30-13:00, Seminar room -1.025 (20.30)

Ill-Posedness of the Third Order NLS Equation with Raman Scattering Term

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We consider the ill-posedness of the Cauchy problem for the third order NLS equation with Raman scattering term on the one dimensional torus.

$$\partial_t u = \alpha_1 \partial_x^3 u + i\alpha_2 \partial_x^2 u + i\gamma_1 |u|^2 u + \gamma_2 \partial_x\big(|u|^2 u\big) - i\Gamma u \partial_x\big(|u|^2\big), \qquad (1)$$
$$t \in [-T,T], \quad x \in \mathbf{T} = \mathbf{R}/2\pi\mathbf{Z},$$
$$u(0,x) = u_0(x), \qquad x \in \mathbf{T}, \qquad (2)$$

where α_j, γ_j $(j = 1,2)$ and Γ are real constants and T is a positive constant. We assume that Γ, $\alpha_1 \neq 0$ and $\frac{2\alpha_2}{3\alpha_1} \notin \mathbf{Z}$. It has been universally used among physicists as a mathematical model for the photonic crystal fiber oscillator (see, e.g., [1]). I show the nonexistence of solutions in the Sobolev space and the norm inflation of the data-solution map at the origin under slightly different conditions, respectively. I also talk about the local unique existence of solutions in the analytic function space. This talk is based on the paper by Kishimoto and Tsutsumi [2].

References

[1] G. Agrawal, *Nonlinear Fiber Optics*, 4th edition, Elsevier / Academic Press, Burlington, 2007.

[2] N. Kishimoto and Y. Tsutsumi, Ill-posedness of the third order NLS equation with Raman scattering term, *preprint*, arXiv: 1706.09111v1 [math.AP].

Minisymposium 14 Tuesday, 15:00-15:30, Seminar room -1.025 (20.30)

Nonlinear profile decompositions and scattering for a NLS-ODE model

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In this talk we discuss a joint paper with Maeda. We consider a Hamiltonian system combining a nonlinear Schrödinger equation (NLS) coupled to an ordinary differential equation. It is a simplified model of the NLS around soliton solutions. Following a recent result by Nakanishi, who initiated the theory of profile decompositions in systems involving a mixture of a continuous and a discrete coordinate, we show scattering of L^2 small H^1 radial solutions. The proof is based on Nakanishi's framework. However, compared to Nakanishi, here the equation of the continuous mode contain some forcing terms that depend only on the discrete mode, and that makes it difficult to prove the usual decoupling between the profiles.

References

[1] S. Cuccagna and M. Maeda, On nonlinear profile decompositions and scattering for a NLS-ODE model, arXiv:1701.02849.

[2] K.Nakanishi, Global dynamics below excited solitons for the nonlinear Schrödinger equation with a potential, *J. Math. Soc. Japan*, in press.

Minisymposium 14 **Tuesday, 15:30-16:00, Seminar room -1.025 (20.30)**

Initial-Boundary Value Problems for the Reaction-Diffusion Equation

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The reaction-diffusion equation supplemented with Dirichlet boundary conditions either on the half-line or on a finite interval is shown to be locally well-posed in the sense of Hadamard for data in Sobolev spaces H^s. In both domains, the proof takes advantage of a novel solution formula for the forced linear heat equation obtained via the unified transform method of Fokas. This formula provides the basis for setting up a Picard iteration scheme for the nonlinear problem and for establishing the various linear estimates required for showing local well-posedness via a contraction mapping argument. In this latter context, interesting and somewhat unexpected estimates are derived not only at the level of the initial-boundary value problems but also in connection with the linear heat initial value problem.

Minisymposium 14 **Wednesday, 11:30-12:00, Seminar room 1.067 (20.30)**

Inverse scattering transform for the integrable nonlocal nonlinear Schrödinger equation

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[2] *Department of Applied Mathematics, University of Colorado, Boulder, CO*

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A nonlocal nonlinear Schrödinger (NLS) equation was recently introduced in [1,2,3] and shown to be an integrable infinite dimensional Hamiltonian evolution equation. In this talk we present a detailed study of the inverse scattering transform of this nonlocal NLS equation. The direct and inverse scattering problems are analyzed. Key symmetries of the eigenfunctions and scattering data and conserved quantities are discussed. The inverse scattering theory is developed by using a novel left-right Riemann – Hilbert problem. The Cauchy problem for the nonlocal NLS equation is formulated and methods to find pure soliton solutions are presented; this leads to explicit time-periodic one and two soliton solutions. A detailed comparison with the classical NLS equation is given and brief remarks about nonlocal versions of the modified Korteweg - de Vries and sine-Gordon equations are made.

References

[1] M. J. Ablowitz and Z. H. Musslimani, *Physical Review Letters*, textbf110, 064105 (2013).

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Minisymposium 14 **Wednesday, 12:00-12:30, Seminar room 1.067 (20.30)**

Stability of solitary waves in $\mathcal{PT}$-symmetric systems

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We consider stability of solitary waves in the $\mathcal{PT}$-symmetric systems by using ideas from the theory of Hamiltonian systems.

For the continuous nonlinear Schrödinger equation, we introduce *Krein quantity* for isolated neutrally stable eigenvalues, which is real and nonzero for simple eigenvalues but it vanishes if two simple eigenvalues coalesce into a defective eigenvalue. A necessary condition for bifurcation of unstable eigenvalues from the defective eigenvalue is proved. This condition requires the two simple eigenvalues before the coalescence point to have *opposite* Krein signatures.

For the discrete Dirac model, we introduce a class of $\mathcal{PT}$-symmetric lattices which enjoys a cross-gradient Hamiltonian structure. Solitary waves are saddle points of the action functional, which are located between the continuous bands of positive and negative energy. Despite not rendering the energy minima, the solitary waves are shown to be nonlinearly stable in the long but finite time evolution by using the Lyapunov method.

The presentation is based on the recent joint works with Alexander Chernyavsky [1,2].

References

[1] A. Chernyavsky and D.E. Pelinovsky, *Long-time stability of breathers in Hamiltonian PT-symmetric lattices*, J. Phys. A: Math. Theor. **49**, 475201 (20pp) (2016).

[2] A. Chernyavsky and D.E. Pelinovsky, *Krein signature for instability of $\mathcal{PT}$-symmetric states*, Physica D, in press (2018).

Minisymposium 14 **Wednesday, 12:30-13:00, Seminar room 1.067 (20.30)**

Discrete breathers in PT-Symmetric metamaterials

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In this work we investigate a one-dimensional parity-time (PT)-symmetric magnetic metamaterial consisting of split-ring dimers having both gain and loss. Employing a Melnikov analysis we study the existence and persistence of localized travelling waves and study their linear stability. We find conditions under which the homoclinic orbits persist using the homoclinic Melnikov Method. Our analytical results are found to be in good agreement with direct numerical computations.

References

[1] M. Agaoglou, M. Feckan, M. Pospisil, V.M. Rothos, H. Susanto *Wave Motion* **76** (2018), pp. 9-18.

Minisymposium 14 Wednesday, 15:00-15:30, Seminar room 1.067 (20.30)

On orbital stability of ground states for finite crystals in fermionic Schrödinger–Poisson model

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We consider the Schrödinger–Poisson–Newton equations for finite crystals under periodic boundary conditions with one ion per cell of a lattice. The electron field is described by the N-particle Schrödinger equation with antisymmetric wave function. Our main results are (i) the global dynamics with moving ions, and (ii) the orbital stability of periodic ground state under a novel Jellium and Wiener-type conditions on the ion charge density [1]. Under Jellium condition both ionic and electronic charge densities of the ground state are uniform.

The earlier result with one particle Schrödinger equation was obtained in [2].

References

[1] A. Komech, E. Kopylova, On orbital stability of ground states for finite crystals in fermionic Schrödinger–Poisson model, *SIAM J. Math. Analysis* **50** (2018), no. 1, pp. 64–85.

[2] A. Komech, E. Kopylova, On stability of ground states for finite crystals in the Schrödinger–Poisson model, *J. Math. Phys.* **58** (2017), no. 3, 031902 (18pp).

Minisymposium 14 Wednesday, 15:30-16:00, Seminar room 1.067 (20.30)

Strong instability of standing waves for nonlinear Schrödinger equations with potential

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We study the instability of standing wave solutions $e^{i\omega t}\phi_\omega(x)$ for L^2-supercritical nonlinear Schrödinger equation with an attractive potential

$$i\partial_t u = -\Delta u + V(x)u - |u|^{p-1}u, \quad (t,x) \in \mathbb{R} \times \mathbb{R}^N, \tag{1}$$

where $p > 1+4/N$ and $p < 1+4/(N-2)$ if $N \geq 3$, ω is a real parameter, and ϕ_ω is a ground state of the corresponding stationary problem. We first review some results on strong instability by blowup of standing wave solutions $e^{i\omega t}\phi_\omega(x)$ for (1) with an attractive delta function potential $V(x) = -\delta(x)$ in one space dimension (see [2]), and with a harmonic potential $V(x) = |x|^2$ (see [1]). Then, we introduce our recent developments in this problem.

References

[1] M. Ohta, Strong instability of standing waves for nonlinear Schrödinger equations with harmonic potential, *Funkcial. Ekvac.* **61** (2018), pp. 135–143.

[2] M. Ohta and T. Yamaguchi, Strong instability of standing waves for nonlinear Schrödinger equations with a delta potential, *RIMS Kôkyûroku Bessatsu* **B56** (2016), pp. 79–92.

Contributed Talks

Contributed Talk Monday, 11:45-12:15, Seminar room 2.067 (20.30)

Fractional derivatives in models of wave propagation in viscoelastic media

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Fractional calculus is a powerful tool for modeling phenomena arising in diverse fields such as mechanics, physics, engineering, economics, finance, medicine, biology, chemistry, etc. It deals with derivatives and integrals of arbitrary real (or even complex) order, thus extending capabilities of the classical calculus, but also introducing novelties in theoretical and applied research. The focus of this talk is on the investigation of waves in viscoelastic media through the constitutive equation containing fractional derivatives of various type.

So far, the classical wave equation has been generalized for the case of viscoelastic materials by the use of fractional derivatives of constant real order. Our most recent study introduces a distributed order fractional model to describe wave propagation in viscoelastic infinite media, and examines existence and uniqueness of fundamental solutions for the corresponding generalized Cauchy problem. Some particular cases of distributed order fractional constitutive stress-strain relations will be presented in more details, as well as numerical experiments, in order to illustrate theoretical results.

This talk is based on joint work with Lj. Oparnica and D. Zorica.

Contributed Talk Monday, 12:15-12:45, Seminar room 2.067 (20.30)

Long-Time Existence of Solutions to Nonlocal Nonlinear Wave Equations with Nonsmooth Kernels

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We consider the Cauchy problem defined in [1] for a general class of nonlocal wave equations appearing in the nonlocal elasticity theory of solids. We prove a long time existence result for the nonlocal wave equations with power-type nonlinearities if the initial data is sufficiently smooth. For the existence proof of the linearized system, we follow the standard hyperbolic approach in [2]. The energy estimates for the linearized problem involve loss of derivatives and we use the Nash-Moser theorem proved in [3] as the main technical tool. We discuss some particular cases, including the limiting case where the kernel function is the Dirac measure and the nonlocal equation reduces to the governing equation of one-dimensional classical elasticity theory.

References

[1] N. Duruk, H. A. Erbay and A. Erkip, Global Existence and Blow-up for a Class of Nonlocal Nonlinear Cauchy Problems Arising in Elasticity, *Nonlinearity* **23** (2010), pp. 107–118.

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Contributed Talk Monday, 12:45-13:15, Seminar room 2.067 (20.30)

Critical points in Strichartz functional

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We study a pair of infinite dimensional dynamical systems naturally associated with the study of minimizing/maximizing functions for the Strichartz inequalities for the Schrödinger equation. One system is of gradient type and the other one is a Hamiltonian system. For both systems, the corresponding sets of critical points, their stability, and the relation between the two are investigated. By a combination of numerical and analytical methods we argue that the Gaussian is a maximizer in a class of Strichartz inequalities for dimensions one, two and three. The argument reduces to verification of an apparently new inequality involving binomial coefficients.

Contributed Talk Monday, 15:00-15:30, Seminar room 2.067 (20.30)

Radiation conditions for periodic potentials

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The stationary Schrödinger operator $H = -\Delta + V(x)$ on $L^2(\mathbb{R}^d), d \geq 2$ with a periodic potential V has been intensively studied for many years by both mathematicians and physicists. It plays an important role in solid state physics and has applications to photonic crystal, metameterial, carbon nanostructure, and topological insulator. The main tool for periodic operators, Floquet-Bloch-Gelfand transform, reduces the study of a periodic operator on whole space $\mathbb{R}^d$ to the study of this operator on a bounded domain with various boundary conditions. Perturbation theory exhibits the band gap structure of the spectrum of the operator H. The main purpose of this talk is to study radiation conditions for the corresponding periodic Schrödinger equation

$$-\Delta u(x) + V(x)u(x) - \lambda u(x) = f(x), \qquad (1)$$

where V is a smooth real-valued function on $\mathbb{R}^d$, periodic with respect to the integer lattice $\mathbb{Z}^d$, $f \in C_c^\infty(\mathbb{R}^d)$ and λ is the spectral parameter in the spectrum of H. I will present radiation conditions and related geometric conditions on the Fermi surface for the equation (1).

Contributed Talk **Monday, 15:30-16:00, Seminar room 2.067 (20.30)**

The Faraday cage effect in a tri-dimensionnal plane periodic structure

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In this talk, we consider the scattering of electromagnetic waves (time-harmonic Maxwell's equations) through a thin periodic plane perforated layer made of equi-spaced perfectly conducting bodies. The size of the obstacles and the distance between two consecutive ones are of the same order of magnitude ε, ε being small. We show that the limit as ε tends to 0 of the solution depends on the topology of the obstacles constituting the periodic layer: if the thin layer is made of disjoint obstacles, it disappears at the limit. By contrast, a thin layer made of a mesh of wires acts as a barrier (shielding effect). Our result, that shares similarities with those obtained in [3] in a full 3D periodic configuration, extends [1]-[2].

References

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Contributed Talk **Monday, 16:30-17:00, Seminar room 2.067 (20.30)**

Spectral estimates for Dirichlet Laplacian on twisted tubes.

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In first part of the talk we investigate Dirichlet Laplacian in a straight twisted tube of a non-circular cross section with a local perturbation of the twisting velocity. It is known that the essential spectrum covers the half- line. Under some additional assumptions on the twisting velocity perturbation the non-emptiness of the discrete spectrum is also guaranteed $\{P.Exner, H.Kovarik, 2005\}$.

In second part of the talk we study Dirichlet Laplacian on a straight tube but already with twisting velocity exploding to infinity. Then the spectrum is becoming purely discrete $\{D.Krejcirik, 2015\}$.

In both cases we investigate the upper bounds for eigenvalue momens.

This is a joint work with Pavel Exner and Andrii Khrabustovskyi.

Contributed Talk	Monday, 17:00-17:30, Seminar room 2.067 (20.30)

A Global div-curl-Lemma for Mixed Boundary Conditions in Weak Lipschitz Domains and a Corresponding Generalized A_0^*-A_1-Lemma in Hilbert Spaces

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We prove global (and hence also local) versions of the so called div-curl-lemma, also known as compensated compactness, for bounded weak Lipschitz domains in 3D with weak Lipschitz interfaces and for mixed boundary conditions. Generalizing our results using an abstract Hilbert space setting, we are able to show corresponding results to hold in arbitrary dimensions as well as for various differential operators for plenty applications. The proofs are quite simple, using as crucial tools Hilbert complexes and related compact embeddings, and reveal the core of all div-curl-lemmas.

Our general result reads as follows: Let $A_0 : D(A_0) \subset H_0 \to H_1$, $A_1 : D(A_1) \subset H_1 \to H_2$ be two densely defined, closed, and possibly unbounded linear operators on three Hilbert spaces H_1, H_2, H_3 with the complex property $A_1 A_0 = 0$. (Think of $A_0 = \text{grad}$ and $A_1 = \text{curl}$ or $A_0 = \text{curl}$ and $A_1 = \text{div}$ with possibly even mixed homogeneous boundary conditions.)

Theorem Let $D(A_1) \cap D(A_0^*) \hookrightarrow H_1$ be compact. Moreover, let $(x_n) \subset D(A_1)$ as well as $(y_n) \subset D(A_0^*)$ be two $D(A_1)$-bounded resp. $D(A_0^*)$-bounded sequences. Then there exist $x \in D(A_1)$ and $y \in D(A_0^*)$ and two subsequences, again denoted by (x_n) and (y_n), such that (x_n) resp. (y_n) converges weakly to x resp. y in $D(A_1)$ resp. $D(A_0^*)$ and

$$\langle x_n, y_n \rangle_{H_1} \to \langle x, y \rangle_{H_1}.$$

Contributed Talk	Monday, 17:30-18:00, Seminar room 2.067 (20.30)

Geometrical approximations of Schrödinger operators with point interactions

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In this talk we address the problem of an approximation of solvable models in quantum mechanics. *Solvable models* describe the motion of a particle in a potential being supported at a discrete set. The term “solvable” reflects the fact that their mathematical and physical quantities (spectrum, eigenfunctions, etc.) can be determined explicitly. Such models are also called *point interactions*. We refer to [1] for a comprehensive introduction to this topic.

One of the main problems arising in the theory of solvable models is their approximations by more “realistic” ones. In the talk we address the question of approximation of the called δ and δ'-interactions using geometrical tools, namely, by the Neumann Laplacians on thin domains with waveguide geometry. For the underlying operators we establish (a kind of) norm resolvent convergence and the Hausdorff convergence of their spectra. To approximated δ-interactions we use waveguides with attached “room-and-passage” bumps, while for δ'-interactions we utilize waveguides consisting of two thin straight tubular domains connected through a tiny window.

References

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Contributed Talk	Tuesday, 11:30-12:00, Seminar room 0.014 (20.30)

Mind the gap - a splitting approach to highly oscillatory differential equations

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We consider the second order semilinear differential equations of the form

$$q'' = -\Omega^2 q + g(q),$$

where Ω is symmetric and positive definite matrix with large norm. The nonlinear function $g(q)$ is assumed to be "nice".

Trigonometric integrators are able to solve such problems using time steps τ which are much larger than $\|\Omega\|^{-1}$. For linear problems we showed in [2] that the interpretation of trigonometric integrators as splitting methods leads to a new convergence proof. We now present a way to transfer this technique to semilinear problems. Compared to results known in the literature [1], the new ansatz provides different (sufficient) conditions on the filter functions used in the trigonometric integrator. This provides a new perspective on the analysis of highly oscillatory problems which can hopefully be extended to a larger class of such problems.

References

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Contributed Talk	Tuesday, 12:00-12:30, Seminar room 0.014 (20.30)

Splitting Methods for Plasmonic Nanostructures

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In this talk we consider different time integration schemes for plasmonic nanostructures, especially splitting methods. Time integration is a big challenge, since simulations have to run over very long time periods to gain physical relevant results.

We describe these structures by a linearized hydrodynamic Drude model, which consists of linear Maxwell's equations coupled with a damped wave equation. The space discretization is done via a standard discontinuous Galerkin method with central fluxes.

Primarily we investigate several splitting ansatzes and recombinations of the equations and compare them to methods proposed in literature, e.g. to a modified leapfrog method [1] and to optimized low-storage Runge-Kutta methods [2]. For these schemes we show that for some settings they are more efficient than the low-storage Runge-Kutta methods or the modified leapfrog method.

References

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Contributed Talk **Tuesday, 12:30-13:00, Seminar room 0.014 (20.30)**

A splitting approach for the magnetic Schrödinger equation

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The Schrödinger equation in the presence of an external electromagnetic field is an important problem in computational quantum mechanics. It also provides a nice example of a differential equation where it is advantageous to employ operator splitting and to split the flow into three physically different parts.

The treatment of the advection part requires special care in order not to lose the conservation properties of the scheme. We discuss several options. Numerical examples in one, two and three space dimensions show that the method of characteristics coupled with a non-equispaced fast Fourier transform (NFFT) provides a fast and reliable technique for achieving mass conservation at the discrete level.

This is joint work with M. Caliari (Verona) and A. Ostermann (Innsbruck).

References

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Contributed Talk **Tuesday, 15:00-15:30, Seminar room 0.014 (20.30)**

A unified error analysis for non-conforming space discretizations of wave-type equations

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In this talk we present a unified error analysis for non-conforming space discretizations of linear wave equations in time-domain. Our abstract framework considers wave equations as first-order evolution equations in Hilbert spaces and allows us to derive a priori error bounds in terms of interpolation, data and conformity errors of the space discretization method.

Our results significantly ease the proof of new convergence rates. We demonstrate this for an isoparametric finite element discretization of the wave equation with dynamic boundary conditions in a smooth domain Ω. Such discretizations are non-conforming, since triangulations of Ω lead to a computational domain $\Omega_h \neq \Omega$. Moreover, the unified error analysis is able to reproduce known convergence rates. For instance, it covers discontinuous Galerkin methods for Maxwell's equations and mass lumped finite elements for the scalar wave equation.

We dedicate a part of this talk to the modular nature of our framework. This feature enables us to derive error bounds for full discretizations by a simple combination of space and time error estimates.

References

[1] D. Hipp, M. Hochbruck and C. Stohrer, Unified error analysis for non-conforming space discretizations of wave-type equations *CRC 1173-Preprint* **2017/29**, 2017.

Contributed Talk **Tuesday, 15:30-16:00, Seminar room 0.014 (20.30)**

Linearly implicit time integration of semilinear wave equations with dynamic boundary conditions

Marlis Hochbruck[1], **Jan Leibold**[1,*]
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In this talk we present a linearly implicit time integration scheme for semilinear wave equations with a non-stiff nonlinearity. Such methods treat the (stiff) linear part of the differential equation implicitly and the nonlinear part explicitly. Thus they require only the solution of one linear system of equations in each time step. We investigate the stability of the scheme and show a second order error bound.

As an application, we consider a finite element discretization of a semilinear acoustic wave equation with dynamic boundary conditions as in [2]. First we sketch the proof of a full discretization error bound using the unified error analysis from [1]. Afterwards we present numerical experiments which show that the linearly implicit method is competitive to standard time integration methods like the Crank-Nicolson or the leapfrog scheme.

References

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Contributed Talk **Tuesday, 16:30-17:00, Seminar room -1.025 (20.30)**

Coupling Problems of Wave-type Equations

Sarah Eberle[1,*]
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In this talk we analyse coupling problems of wave-type equations, e.g., the coupling of interior-exterior problems of the elastodynamic wave equation in 3D [1]. Thus, we introduce transparent boundary conditions, which are non local in space and time. The key issue is a coercivity property of a Calderón operator for the elastic Helmholtz equation, which is valid for all complex frequencies in a half-plane. We prove the stability and convergence of the numerical methods, where we apply finite elements (FEM) and leapfrog in the interior and boundary elements (BEM) and convolution quadrature (CQ) on the boundary. Finally, we present numerical examples for a non-convex domain.

References

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Contributed Talk Tuesday, 17:00-17:30, Seminar room -1.025 (20.30)

Transparent boundary conditions for the KdV equation

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We propose a way to treat the transparent boundary conditions for the linearized Korteweg–de Vries (KdV) equation with space dependent coefficients:

$$u_t + a(x)u_x + b(x)u_{xxx} = 0.$$

We follow a splitting strategy in order to divide the full equation into its dispersive part $u_t + b(x)u_{xxx} = 0$ and its transport part $u_t + a(x)u_x = 0$. The transparent boundary conditions are then derived in a full–discrete setting using the Crank–Nicolson and explicit Euler finite different schemes for the dispersive and transport equation respectively. Numerical simulations are presented that illustrate the theoretical results.

Contributed Talk Tuesday, 17:30-18:00, Seminar room -1.025 (20.30)

Numerical simulation of rf-SQUIDs

Marlis Hochbruck[1], **Bernhard Maier**[1,*], **Marvin Müller**[2], **Carsten Rockstuhl**[2]
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We study the interaction of electromagnetic waves with rf-SQUIDs aligned on a thin film [1]. This yields a system of Maxwell's equations coupled with an anharmonic oscillator via a jump condition for the normal derivative at the interface. With our main interest being the calculation of the reflection and transmission coefficients of the film, we introduce transparent boundary conditions [2]. This drastically reduces the computational effort as the spatial grid for the one-dimensional numerical examples only contains 2 cells.

In this talk, we show well-posedness using [3] for a first order reformulation of this system. We further discuss the discretization with finite elements in space and a Strang splitting in time. Finally, we present an error analysis and numerical results.

References

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Contributed Talk	Tuesday, 16:30-17:00, Seminar room 0.014 (20.30)

The damped wave equation with unbounded damping

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Abstact: We consider the linear damped wave equation on possibly unbounded domains with the damping being allowed to become unbounded at infinity and we analyze newly arising phenomena. We show the generation of a contraction semigroup, the relation of spectra of the equation generator and the associated quadratic operator function (having the form of Schrödinger operator with a complex potential), the convergence of non-real eigenvalues in asymptotic regime of an infinite damping on a subdomain and investigate both non-real eigenvalues and the negative essential spectrum. The presence of the latter turns out to be a robust effect that cannot be easily canceled by adding a positive potential and so the exponential estimate of the semigroup is often lost. Finally, we investigate spectral instablibity (pseudospectrum) of the semigroup generator. The analytic results will be illustrated by several examples in various dimensions.

References

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Contributed Talk	Tuesday, 17:00-17:30, Seminar room 0.014 (20.30)

On the Energy Rate of Decay for the linear Damped Klein Gordon Equation on Unbounded Domain.

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We study the time decay of the energy for the damped Klein-Gordon equation. We give an explicit necessary and sufficient condition on the continuous damping functions $\lambda \geq 0$ for which the energy $E(t) = \int_{-\infty}^{\infty} |u_x|^2 + |u|^2 + |u_t|^2 dx$ decays exponentially, whenever $(u(0), u_t(0)) \in H^2(\mathbb{R}) \times H^1(\mathbb{R})$.

Contributed Talk **Tuesday, 17:30-18:00, Seminar room 0.014 (20.30)**

On the Stability of Traveling Wave Solutions to the Fornberg-Whitham Equation

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An alternate model

$$u_t + \frac{3}{2}uu_x + K * u = 0$$

for the wave motion at the surface of a fluid, has been proposed in [1, 2] to describe the qualitative properties of water waves such as wave breaking or the existence of greatest wave. Special choices of the kernel K lead to equations like Whitham equation, Fornberg-Whitham (FW) equation and etc. FW equation is not only capable of modeling some of the important properties of water waves but also has explicit solutions in several forms.

In this talk, we discuss the stability of traveling wave solutions of FW equation. We also numerically discuss the effects of different kernel functions on the stability.

References

[1] G. B. Whitham,*Linear and Nonlinear Waves*, 1st edition, Wiley, New York, 1974.

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Contributed Talk **Wednesday, 11:30-12:00, Seminar room 2.066 (20.30)**

Analysis of the hp-version of a first order system least squares method for the Helmholtz equation.

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We consider a first order system least squares method for the Helmholtz problem

$$\begin{aligned} -\Delta u - k^2 u &= f \quad \text{in } \Omega, \\ \partial_n u - iku &= g \quad \text{on } \partial\Omega. \end{aligned}$$

Extending the results of [1], we show *a priori* estimates in the L^2-norm for the least squares method that is explicit in h and p. These estimates are valid under the scale resolution condition that kh/p is sufficiently small and $p/\log k$ is sufficiently large. Our key refinement over [1] is a wavenumber explicit regularity estimate of a suitable dual problem, which in turn allows for optimal p-dependence in the *a priori* estimate. As a tool, which is of independent interest, we develop approximation operators in Raviart-Thomas spaces with optimal (in h and p) approximation rates simultaneously in L^2 and $H(\mathrm{div})$.

References

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Contributed Talk	Wednesday, 12:00-12:30, Seminar room 2.066 (20.30)

Parallel HPC Solution of the Helmholtz Equation with Controllability Methods

M. J. Grote[1], **F. Nataf**[2], **J. H. Tang**[1,*], **P.-H. Tournier**[2]
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The exact controllability approach [1] reformulates the Helmholtz equation in the time domain and seeks the time-harmonic solution of the corresponding wave equation by iteratively minimizing an appropriate penalty functional. Then each conjugate gradient iteration solely relies on standard numerical algorithms, which are inherently parallel and robust against higher frequencies. In [2], new penalty functionals were introduced which extend the controllability approach to general boundary value problem governed by the Helmholtz equation. Here, we study the parallel performance of controllability methods using `FreeFem++` [3] on massively parallel HPC architectures for large-scale Helmholtz problems.

References

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Contributed Talk	Wednesday, 12:30-13:00, Seminar room 2.066 (20.30)

Efficient stochastic sparse photoacoustic solver using ray tracing

Francesc Rul·lan[1,*], **Marta M. Betcke**[1]
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In this talk we explore the use of high frequency asymptotic approximation to the solution of the wave equation in photoacoustic tomography (PAT). We consider ray tracing, a technique that involves solving the wave equation along the trajectories of a Hamiltonian system. We propose ray tracing based algorithms for the solution of the forward and adjoint PAT problems in a 3D scenario. We evaluate the quality of the ray tracing solutions to both the forward and adjoint problems against a full wave solution obtained with the k-space method implemented in k-Wave Toolbox.

The major benefit of ray tracing approach versus the full wave solution, as e.g. obtained with k-Wave, is that the solution at each sensor can be obtained independently and at a much lower cost than the full k-Wave solution. This flexibility affords an advantage in the construction of an approximate solver for use in iterative schemes. We combine the fast partial forward and adjoint ray tracing operators (obtained when taking a single sensor) with a stochastic gradient descent scheme and show the results in a 3D domain.

References

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Contributed Talk **Wednesday, 11:30-12:00, Seminar room 3.069 (20.30)**

Dispersion relations of periodic photonic systems with a strong material dispersion

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Periodic photonic structures composed of non-dispersive, lossless materials are very well understood. In contrast, the case of structures involving lossy and strongly dispersive constituents have been studied significantly less. Yet, it is physically relevant e.g. in the form of hyperbolic metamaterials or strong coupling in periodic media. Here, we report on our recent[1] progress in developing an adjoint mode formalism for the calculation of the complex bandstructure derivative $(\partial\omega)/(\partial\mathbf{k})$ (complex group velocity) and the density of transverse optical states. Our exact expressions hold for 3D periodic arrays of materials with arbitrary dispersion properties and in general need to be evaluated numerically. We present the overall mathematical framework and as an example discuss the analytically accessible case of structures composed of lossless dielectrics perturbed by one sharp Lorentzian material resonance.

References

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Contributed Talk **Wednesday, 12:00-12:30, Seminar room 3.069 (20.30)**

Modelling and design of nano-structures: multilayer nanoplasmonics configurations

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Abstract

Nanoplasmonics forms a major part of the field of nanophotonics, which explores how electromagnetic fields can be confined over dimensions on the order of or smaller than the wavelength. Here, we present an integral-equation formulation of the mathematical model that delivers accurate solutions in small computational times for surface plasmons coupled by periodic corrugations of flat surfaces an extension of single layer configurations to a more challenging case; multilayered configurations. The new configuration is composed of a thin layer of a metal (gold, silver, etc.) with depth larger than skin depth of the material, buried into different epoxies on top (glass/polymer substrate) and the bottom (liquid/water/blood) both extend infinitely above and below the surface. Some details of the numerical implementation and the results of a few numerical experiments are also given

Contributed Talk	Wednesday, 12:30-13:00, Seminar room 3.069 (20.30)

Characterization of metamaterials beyond a local response

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Metamaterials (MM) are a novel class of artificial materials that can control electromagnetic fields in a way inaccessible with ordinary materials. Studying their electromagnetic response is tightly linked to the introduction of effective material properties. Introducing local material parameters for a MM requires the operational wavelength to be much longer with respect to the typical length scale of the microscopic structure. However, for most MMs this does not apply and the operational wavelength is only slightly longer than their typical length scale. Then, local material properties turned out to be insufficient to adequately capture the electromagnetic response.

Here, we go beyond the local approximation and study nonlocal material properties. We analyze, by means of a comparison to an actual MM, both dispersion relation of the bulk and reflection and transmission coefficients from a slab and show that retaining nonlocal parameters enlarges the parameter space, i.e., wavelengths and angles of incidence, where homogenization is meaningful. This significant improvement confirms the importance of considering nonlocality for a realistic prediction the electromagnetic response of a MM.

References

[1] K. Mnasri, A. Khrabustovskyi, C. Stohrer, M. Plum, and C. Rockstuhl Phys. Rev. B **97**, 075439 (2018)

Contributed Talk	Wednesday, 15:00-15:30, Seminar room 2.066 (20.30)

Uniformly accurate methods for Klein-Gordon-type equations

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Solving Klein-Gordon-type equations in the non-relativistic limit regime is numerically very delicate as the solution becomes highly oscillatory in time. In particular, classical numerical schemes suffer from severe time step restrictions. The aim of the talk is to present a new class of uniformly accurate numerical methods (see [1,2,3]) which yield good results in highly oscillatory non-relativistic as well as in purely relativistic regimes. Our novel class of time integrators is based on a new idea of *twisted variables*. Numerical experiments confirm the favorable error behavior of our uniformly accurate methods.

References

[1] S. Baumstark, E. Faou and K. Schratz, Uniformly accurate exponential-type integrators for Klein-Gordon equations with asymptotic convergence to the classical NLS splitting, *Math. Comp.* **87** (2018), pp. 1227–1254.

[2] S. Baumstark, G. Kokkala and K. Schratz, Asymptotic consistent exponential-type integrators for Klein-Gordon-Schrödinger systems from relativistic to non-relativistic regimes, *ETNA* **48** (2018), pp. 63–80.

[3] S. Baumstark and K. Schratz, Uniformly accurate oscillatory integrators for the Klein-Gordon-Zakharov system from low- to high-plasma frequency regimes, *preprint* (2018).

Contributed Talk Wednesday, 15:30-16:00, Seminar room 2.066 (20.30)

Efficient Numerical Schemes for Highly Oscillatory Klein–Gordon and Dirac type Equations

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Klein–Gordon and Dirac equations physically describe the motion of relativistic particles. The construction of efficient numerical time integration schemes for solving these equations in the nonrelativistic limit regime, i.e. when the speed of light c formally tends to infinity, is numerically very delicate, as the solution becomes highly-oscillatory in time. In order to resolve the oscillations, standard time integrations schemes require severe restrictions on the time step $\tau \sim c^{-2}$ depending on the small parameter c^{-2} which leads to high computational costs.

In my talk, I will present numerical techniques based on [1] for efficiently solving these highly oscillatory systems without any time step restriction by exploiting their inherent time-oscillatory structure. We carry out the construction of these schemes by filtering out the highly oscillatory phases (in time) explicitly, which allows us to break down the numerical task to solving slowly oscillatory Schrödinger-type systems.

References

[1] S. Baumstark, E. Faou and K. Schratz, Uniformly accurate exponential-type integrators for Klein-Gordon equations with asymptotic convergence to the classical NLS splitting, *Math. Comp.* **87**(2018), pp. 1227–1254.

Contributed Talk Wednesday, 16:30-17:00, Seminar room 2.066 (20.30)

Post-processed Galerkin approximation of improved order for wave equations

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We introduce and analyze a post-processing for a family of variational space-time approximations to wave problems; cf. [1]. The discretization in space and time is based on continuous finite element methods. The concepts can be applied similarly to discontinuous in time and space approximations of wave equations. The post-processing lifts the fully discrete approximations in time from continuous to continuously differentiable ones. The order of convergence of the discretization in time is increased by one which can be used, for instance, for a-posteriori error control. The convergence behavior is shown by proving error estimates of optimal order in various norms. A bound of superconvergence at the discrete times nodes is also included.

The presented results are a joint work with F. Schieweck, F. A. Radu and U. Köcher. For implementational and computational details including the algebraic solver we also refer to [2,3].

References

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[2] U. Köcher, M. Bause, *Variational space-time methods for the wave equation*, J. Sci. Comput., **61** (2014), pp. 424–453.

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Contributed Talk Wednesday, 17:00-17:30, Seminar room 2.066 (20.30)

Reciprocal Mass Matrices for Transient Elastodynamics

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Recently, reciprocal mass matrices, i.e. directly assembled inverse mass matrices, were proposed in [1-4] as an efficient alternative to lumped mass matrices in explicit dynamics. These reciprocal mass matrices possess a mask of the consistent mass or the stiffness matrix and they allow trivial computation of the nodal acceleration from the total force vector. Moreover, free parameters in their construction can be chosen to minimize the spatial dispersion error for low frequency waves and maximize the critical time step as shown in [2]. In this contribution performance of reciprocal mass matrices is analyzed for several practically relevant scenarios.

References

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Contributed Talk Wednesday, 17:30-18:00, Seminar room 2.069 (20.30)

Iterative regularization on a shape-manifold and applications to inverse obstacle scattering

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The identification of boundary curves from indirect measurements is an important and well-studied class of inverse problems. A prominent example are inverse obstacle scattering problems. The usual approach to represent the unknown curve by a positive, periodic radial function suffers from several drawbacks. In particular, regularization terms depend on the choice of the parametrization, and only star-shaped domains can be represented. To overcome these drawbacks and restrictions, we use the bending energy as regularization term and a corresponding shape manifold which turns out to be a Hilbert-manifold. This provides us with tools from infinite dimensional Riemannian geometry such as Levi-Civita connection, geodesics, Riemannian exponential map, Riemannian Hessian of a functional, and parallel transport.

We introduce a regularized Newton-type method on this manifold to solve inverse obstacle problems. In each step we first compute a new direction as an element of the tangential space using the derivative of the forward operator and the gradient and the Hessian of the bending energy functional. Then we map this direction back onto the manifold using a retraction, which is a generalization of the exponential map, but computationally more efficient. Our main result is a convergence analysis in terms of the Riemannian distance, i.e. the length of the geodesic connecting the two points on the manifold. We prove convergence rates under source conditions defined in the tangent space. Finally, we present numerical results illustrating the benefits of our approach.

Contributed Talk **Thursday, 11:30-12:00, Seminar room 2.067 (20.30)**

Operator Preconditioning for the Electric Field Integral Equation on Screens

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We consider the electric field integral equation (EFIE) arising from the scattering of time-harmonic electromagnetic waves by a perfectly conducting screen. When discretizing the EFIE by means of Galerkin boundary element methods, one obtains ill-conditioned systems on fine meshes and iterative solvers perform poorly. In order to reduce the number of iterations needed to find a solution, one uses preconditioning. Finding a suitable preconditioner for the case of screens poses some challenges due to the spaces at hand. Moreover, since solutions of the EFIE on screens feature edge singularities, its amenability to adaptive refinement is desirable.

In this presentation, we discuss a new strategy to build a preconditioner for the EFIE on screens using operator preconditioning. For this, we construct a compact equivalent inverse of the EFIE operator on the disk using recently found Calderón-type identities [1]. Furthermore, stable discretization of our preconditioner only requires dual meshes for low-order Lagrangian finite element spaces, which are used to discretize the same energy trace spaces that arise from the Laplacian. As a consequence, our approach allows for non-uniform meshes. Finally, we present some numerical experiments validating our claims.

References

[1] R. Hiptmair, C. Jerez-Hanckes, C. Urzúa-Torres, Closed-Form Inverses of the Weakly Singular and Hypersingular Operators On Disks, *Integral Equations and Operator Theory*, To appear. Available in arXiv :1703.08556 [math.AP].

Contributed Talk **Thursday, 12:00-12:30, Seminar room 2.067 (20.30)**

Scattering of an Electromagnetic Wave by a Perfectly Conducting Obstacle Coated with Thin Layers

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The theory of time harmonic electromagnetic or acoustic waves propagation involves always the structure of the obstacle, which have in most cases thin geometry. In the case of perfectly conducting obstacle coated with a thin dielectric layer some difficulties linked to the numerical simulation appear. To overcome that problem many authors have tried to calculate approximations of an impedance operator (see, e.g., [1,2]) for perfectly conducting obstacle coated by thin shell of dielectric material.
In the present work, we have tried to calculate approximations until the third order for perfectly conducting obstacle coated by two contrasted thin layers of dielectric materials, the approach used is that of Lemrabet *et al.* (see [1]), when they have tried to write the boundary condition on the perfect conductor in terms of Taylor expansion in δ, the thickness of the thin layer.

References

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[2] H. Haddar, P. Joly, *Stability of thin layer approximation of electromagnetic waves scattering by linear and nonlinear coatings*, Journal of Computational and Applied Mathematics 143 (2002), no. 2, 201-236.

Contributed Talk Thursday, 12:30-13:00, Seminar room 2.067 (20.30)

Asymptotic modelling of the wave propagation in presence of an array of Helmholtz resonators

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We will consider the acoustic wave propagation in a waveguide with microperforated absorbers that are arrays of Helmholtz resonators. These microperforated absorbers are used to suppress reflections from walls. Due to the smallness of the periodicity and of the perforations a direct numerical simulation, *e.g.*, with the finite element method (FEM), is only possible for very large costs. Based on homogenization techniques (using both the method of multiscale expansions and the method of matched asymptotic expansions) we find impedance transmission conditions [1], which integrated into numerical methods like the FEM or the boundary element method leads to much lower computational costs. We aim to describe these impedance conditions for the linearized Navier-Stokes equations in a three-dimensional waveguide [2].

References

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[2] K. Schmidt, A. Semin, A. Thöns-Zueva and F. Bake, On impedance conditions for circular multiperforated acoustic liners, *arXiv:1801.04147* (2018).

Contributed Talk Thursday, 15:00-15:30, Seminar room 2.067 (20.30)

Exact complex scalings based on Hardy space infinite elements

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Helmholtz scattering and resonance problems in open domains can be treated using the Hardy space infinite element method. This method is based on the pole condition which characterizes outgoing waves by the poles of their Laplace transforms. Outgoing solutions are approximated in the Laplace domain.

We present an interpretation of Hardy space infinite elements as a truncation-free complex scaling method in space. The discretization matrices can be computed numerically using suitable Gauss-Laguerre quadrature rules. This allows us to deal with non-homogeneous exterior domains.

Similarly to the application of perfectly matched layers in [1] we employ our method to discretize complex scaled Helmholtz resonance problems with frequency dependent scaling functions. The frequency dependency of the scaling function optimizes the complex scaling for all frequencies and reduces the dependency of the approximation on the specific choice of scaling parameter.

References

[1] L. Nannen and M. Wess, Computing scattering resonances using perfectly matched layers with frequency dependent scaling functions, *BIT Numerical Mathematics* (2018)

Contributed Talk **Thursday, 15:30-16:00, Seminar room 2.067 (20.30)**

Numerical experiments of generation and propagation of internal waves using a two-layer non-hydrostatic model

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In the study of oceanic internal waves, ocean density stratification can be well represented as a two-layer fluid system, when pycnocline thickness is relatively small. In this article, a depth integrated model of the two-layer fluid with constant density is considered. A variant of the edge-based non-hydrostatic numerical model is formulated, and the corresponding dispersion relation is derived. The numerical dispersion is shown to agree with the analytical dispersion curves over a wide range of kd, with k is the wave number and d the water depth. The numerical scheme is tested with an interfacial solitary wave propagates over a flat bottom, as well as over a bottom step. A simulation of a laboratory scale internal wave that is generated, propagate, and interact with a triangular barrier is performed. Finally, a real bathymetry case study is also performed, in which a strong current passing through a sill of the Lombok Strait simulates the onset of an interfacial wave.

References

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Contributed Talk **Thursday, 16:30-17:00, Seminar room -1.025 (20.30)**

Dynamic Inverse Problems for Wave Equations

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We are interested in the reconstruction of time- and space dependent coefficients appearing in the wave equation

$$(cu')' + \nu u' - \operatorname{div}(a\nabla u) + qu = f \tag{1}$$

from knowledge of the wave field u which solves this equation in some bounded space-time domain $(0,T)\times\Omega \subset \mathbb{R}^{1+n}$, together with homogeneous initial conditions $u(0) = 0$, $(cu')(0) = 0$ and the Dirichlet boundary condition $u = 0$ on $[0,T]\times\partial\Omega$. We assume $f \in L^2([0,T], L^2(\Omega))$ to be known. In an acoustic setting the coefficients c and a are related to the wave speed and the density of the medium in which the waves propagate. The time dependence of these parameters makes the problem more challenging both theoretically and numerically. The reconstruction of the coefficient q in this setting has been treated in [1]. We present a suitable existence and uniqueness result for the solution of equation (1) and compute the Fréchet derivatives of the solution operator. The inverse problem of identifying any of these coefficients from u is ill-posed in the arising function spaces, therefore we employ an inexact newton method for the numerical inversion and test its performance on artificially generated data.

References

[1] Thies Gerken and Armin Lechleiter, Reconstruction of a Time-Dependent Potential from Wave Measurements, *Inverse Problems* **33** (2017) 094001.

Contributed Talk **Thursday, 17:00-17:30, Seminar room -1.025 (20.30)**

Application of the Floquet-Transform to the Helmholtz Equation and Maxwell Equations on Locally Perturbed Bi-periodic Structures

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We consider time-harmonic scattering problems of electromagnetic and acoustic waves on a bi-periodic inhomogeneous medium which is absorbing on an open set and locally perturbed. Such problems are occurring, e.g., in non-destructive testing methods. For the existence theory, we apply the Bloch-Floquet-transform to reformulate the problem as an equivalent system of coupled variational problems on a bounded domain. This system possesses a unique solution and allows us to construct the solution to the original problem.

Apart from existence theory, the framework also serves for numerical calculation of the solution using the finite element method. We use the solver then to reconstruct the perturbation of the refractive index from artificial noisy data by an inexact newton method.

Contributed Talk **Thursday, 17:30-18:00, Seminar room -1.025 (20.30)**

Data recovery: from limited-aperture to full-aperture

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Many methods have been proposed for inverse scattering problems in the past thirty years. Most of them use full-aperture data, i.e., data of all the observation directions due to all incident directions. However, in many cases of practical interest, it is not possible to measure the full-aperture data. Consequently, only limited-aperture data over a range of angles are available. Various reconstruction algorithms using limited-aperture data have been developed. However, the quality of the reconstructions are not satisfactory. Other than developing methods using limited-aperture data, we take some alternative approaches to recover the data that can not be measured directly[2,3]. Based on these data, using a recent proposed direct sampling method [1], the quality of the shape and location reconstructions will be greatly improved[2,3].

References

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Contributed Talk **Thursday, 16:30-17:00, Seminar room 2.067 (20.30)**

Multiharmonic analysis for nonlinear acoustics with small excitation amplitude

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Acoustic experiments show that the noise absorption by perforated walls differs if small or large acoustic amplitudes are used. In addition, an interaction between different frequencies has been observed. To describe these effects we consider the compressible Navier-Stokes equations in time domain with the nonlinear advection term, which couples velocity and pressure. Here, we consider small amplitudes of the excitation and viscosities like $O(\varepsilon^2)$ with some small parameter ε. Nonlinear acoustics can be described in frequency domain by the multiharmonic analysis applied to the compressible Navier-Stokes equations, which takes into account the frequency ω of the incoming wave and their harmonics $0\omega, 2\omega, 3\omega, \ldots$. With an asymptotic expansion for small viscosities and amplitudes the different harmonic contributions decouple and we describe the solution with a number of frequency domain problems only.

References

[1] A. Thöns-Zueva, K. Schmidt, and A. Semin, Multiharmonic analysis for nonlinear acoustics with different scales, *arXiv:1701.02097*, 2017.

Contributed Talk **Thursday, 17:00-17:30, Seminar room 2.067 (20.30)**

Recovering sound speed and density from the cross covariance function in helioseismology

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The basic observation of time-distance helioseismology is a time serie of the cross-covariance between the line-of-sight velocities at two different locations at the solar surface. Under reasonable assumptions, the cross-covariance function in the frequency domain can be linked to the imaginary part of the Green's function obtained by solving a scalar wave equation. We investigate numerically the reconstruction of density and sound speed from the knowledge of the Green's function as well as its restriction to the imaginary part. We verify the theoretical results of [1,2] and emphasize the difficulties due to the solar application, in particular the exponential decay of the density close to the surface which makes the problem extremely ill-posed as depth increases.

References

[1] A. I. Nachman, Reconstructions from boundary measurements, *Annals of Mathematics* **128** (1988), pp. 531-576.

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Miscellaneous Information

Registration, parallel sessions, coffee breaks and the conference dinner on Thursday evening all take place in building 20.30 (*Kollegiengebäude Mathematik*) at KIT Campus Süd. All plenary talks take place in the *Johann-Gottfried-Tulla-Hörsaal* (Tulla Lecture Theatre) in building 11.40 (*Kollegiengebäude am Ehrenhof*).

The map below shows the location of these buildings, which are only a couple of minutes walking distance (200 metres) away from each other. A full map of the KIT South Campus is available from the KIT web site at https://www.kit.edu/downloads/Campus-Sued.pdf. You can also find the area on Google maps at https://goo.gl/maps/3xngh1Vt3Hv.